

FRB202

1. Find *Laplace Transform* of each of the following functions, indicating the method used and showing the details.

$$(a) f(t) = t \int_0^t e^{-3\tau} \sin(2\tau) d\tau$$

$$(b) f(t) = 16t^2 u(t - 1)$$

2. Find the *Inverse Laplace transform* for each of the following functions, indicating the method used and showing the details.

$$(a) F(s) = \frac{1}{s(s^2 - 9)}$$

$$(b) F(s) = e^{-5s} \tan^{-1}\left(\frac{s}{7}\right)$$

3. Solve the following *initial value problems* (IVP's) using Laplace transform, showing the details.

$$(a) u(t) = 4t^2 - \int_0^t u(t - \tau) e^{-\tau} d\tau$$

$$(b) y'' + 4y' + 5y = 50t, \quad y(0) = 5, \quad y'(0) = -5$$

4. Obtain *Fourier expansion* of the following function, graph the corresponding periodic function.

$$f(x) = \begin{cases} 0 & \text{if } -\pi < x < 0 \\ x^2 & \text{if } 0 < x < \pi \end{cases}$$

5. Solve the *wave equation* $u_{tt} = a^2 u_{xx}$ for the length $L = 1$ with $a^2 = 1$ where,

$$u(0, t) = u(L, t) = u_t(x, 0) = 0, \quad u(x, 0) = x$$

Using separation of variable method.

6. Evaluate each of the following *double integrals*.

$$(a) \int_0^{\frac{\pi}{2}} \int_1^e \frac{\sin y}{x} dx dy$$

$$(b) \int_0^2 \int_0^{2-x} (x + y) dy dx$$

$$(c) \int_0^{0.5} \int_{2x}^1 e^{y^2} dy dx$$

$$(d) \int_{-5}^5 \int_0^{\sqrt{25-x^2}} x^2 y dy dx$$