Benha University   College of Engineering at Banha
Benha Higher Institute of Technology
Department of Mechanical Eng.
Subject: Fluid Mechanics

Model Answer of the Final Exam   Date: Dec./21/2012

اجابة امتحان ميكانيكا الموائع م 201

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Q#1 Q#1 a- i) **Vapor pressure**: Pressure at which liquid boils, or pressure of vapor above liquid at equilibrium state.

Dimensions: $ML^{-1}T^{-2}$ **Units:** Pa, N/m^2, bar, psi

ii) Compressibility of the fluid: It is the volumetric strain per unit pressure change.

Dimensions: $M^{-1}LT^{2}$ **Units:** Pa^{-1}

iii) Dynamic viscosity coefficient: It measures the resistance of the fluid to flow under the effect of shear force.

Dimensions: $ML^{-1}T^{-1}$ **Units:** Kg/m.s, Pa.s, poise,..

iv) Specific weight: It is the weight per unit volume.

Dimensions: $ML^{-2}T^{-2}$ **Units:** N/m^3

v) Laminar flow: Flow at which particles move smoothly in parallel sublayers.
Turbulent flow: Flow at which particles interchange their sublayer' randomly.

No dimensions **No units**

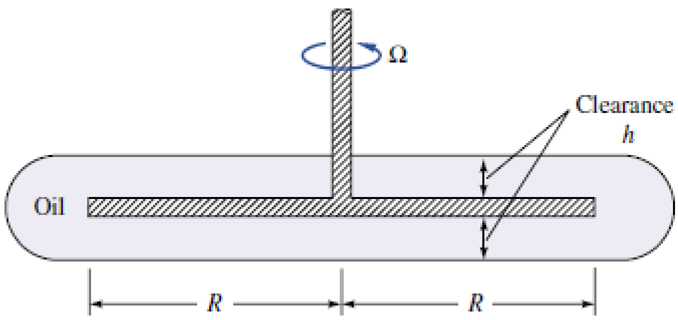
vi) Metacenter: It is the point of intersection of axis of symmetry and line of action of buoyant force, or, point through which line of action of buoyant force is always passing.

No dimensions **No units**

vii) Boundary Layer: It is a thin layer adjacent to the solid surface in which the flow is affected by the solid surface; or It is a thin layer adjacent to the solid surface in which the shear is remarkable.

No dimensions **No units**

b)



We find first the viscous torque (T) required:

$$dT = \tau dA.r = \mu \frac{du}{dy} 2\pi r dr.r = 2\pi\mu \frac{\Omega r}{h} r^2 dr$$

$$T = \frac{2\pi\mu\Omega}{h}\int_0^R r^3 dr = \frac{\pi\mu\Omega R^4}{2h}$$

$$T = 0.1822\ N.m$$

$$power = T.\Omega = 1.4577\ watt$$

1-c)

The weight of the rod is acting at the middle of the rod (C.G).

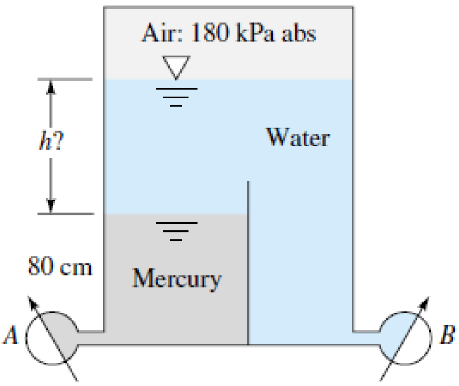
The up thrust R is acting upward at the middle of the immersed portion of the rod. Assume the tension T in the string.

From the equilibrium of forces R = W_{rod} + T $\rightarrow W_{rod} = \rho_{rod} V_{rod} g, R = \rho_{water} x 0.8 V_{rod} g$. Assume that the rod is making an angle θ with the horizontal and taking the moment about the lower end of the rod we get

$$\rightarrow W_{rod} x 2.5 \cos\theta = R x 2 \cos\theta \rightarrow \rightarrow 2.5 \rho_{rod} V_{rod} g = 2 \rho_{water} x 0.8 V_{rod} g \rightarrow \frac{\rho_{rod}}{\rho_{water}} = 0.64$$

The tension in the string T = R- Word = $\frac{\pi}{4}(0.08)^2 x 4 x 1000 x 9.81 - \frac{\pi}{4}(0.08)^2 x 5 x 640 x 9.81 = 39.448 N$

2-a



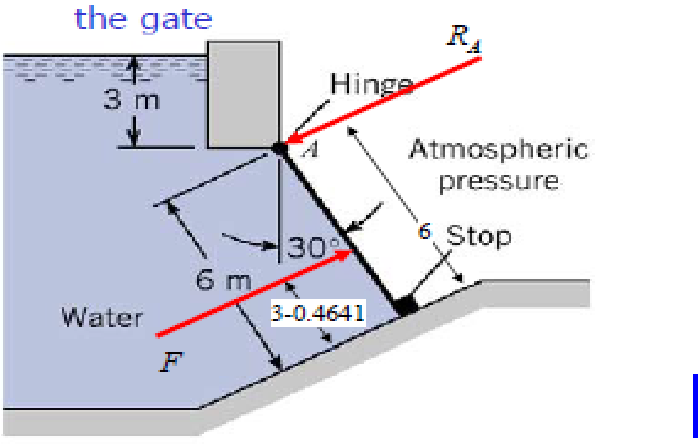
350x1000 = 180x1000 + 9810xh + 0.8x13.6x9810

h = 6.449 m

P_B = 251.115 kPa

2-b)

A rectangular gate (6m × 4m) is hinged at A and supported by a stopper as shown. Find the reaction at the hinge neglecting the weight of the gate



$$F = \bar{p}A = (\gamma \bar{y} \sin\alpha)A$$
$$= 9810*(3+3\cos 30)*(4*6)$$
$$= 1{,}318{,}000\, N$$

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{4*6^3/12}{(6.464*24)}$$
$$= 0.4641\, m$$

$$\sum M = 0$$
$$= 6R_A - (3-0.4641)F$$
$$R_A = \frac{3-0.4641}{6}F$$
$$= (0.42265)1318\, kN$$
$$R_A = 557.05\, kN$$

2-c)

i) Condition of stability of immersed body

The center of buoyancy B should be above the center of gravity CG

ii) Condition of stability of floating body

Meta center M should be above the center of gravity CG

3-a)

b-i) **Steady flow** is the flow whose properties are independent on time.
ii) **Potential flow** is the flow which has zero vorticity.
iii) **Ideal flow** is the non-viscous flow; of the flow which has zero viscosity equal zero.
iv) **Streamline** - An imaginary line in the flow that is everywhere parallel to the local velocity vectors.

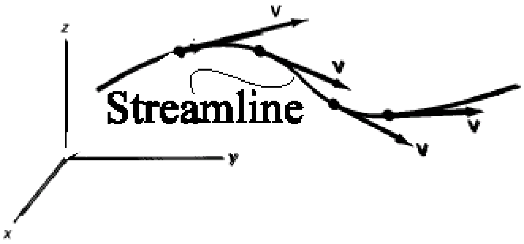


Figure 4

Streakline - An instantaneous line composed of all particles originating from a given point in the flow field; or is the locus of particles which have earlier passed through a prescribed point in space.
A timeline is a set of fluid particles that form a line segment at a given instant of time.
A pathline is the actual path traversed by a given (marked) fluid particle.

3-b) c)) i-

$$\vec{V} = (x^2 - y^2)i - 2xyj$$
$$\vec{V} = ui + vj \rightarrow\rightarrow u = x^2 - y^2, v = -2xy$$
$$\text{the acceleration } \vec{a} = a_x i + a_y j$$
$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \rightarrow\rightarrow a_x = (x^2 - y^2)(2x) + -2xy(\text{-}2y) = 2x^3 + 2xy^2$$
$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \rightarrow\rightarrow a_y = (x^2 - y^2)(\text{-}2y) + -2xy(\text{-}2x) = 2y^3 + 2yx^2$$
$$a_x = 80, a_y = 160 \rightarrow \vec{a} = 80i + 160j$$

ii) The flow will represent a physical flow if $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, for this flow we have $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2x - 2x = 0$

$\therefore$ The flow is a physical flow.

iii) The vorticity is given by : $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -2y + 2y = 0$

iv) $u = x^2 - y^2 = \frac{\partial \psi}{\partial y} \rightarrow \psi = x^2 y - \frac{y^3}{3} + f(x)$

$$v = -2xy = -\frac{\partial \psi}{\partial x} \rightarrow \psi = x^2 y + g(y)$$
$$\therefore \psi = x^2 y - \frac{y^3}{3}$$

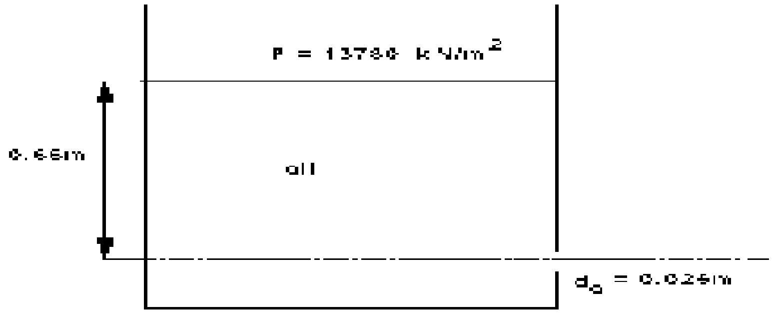
The equation of the streamline passing through the point (1,2) is given by: $3x^2 y - y^3 = -2$

3-c

Assumptions for derivation of Bernoulli equation

1- Steady flow
2- Flow along streamline
3- Ideal (non-viscous) flow
4- Incompressible flow
5- No shaft work
6- No heat transfer.

4-a)



From the question

$$\sigma = 0.9 = \frac{\rho_o}{\rho_w}$$
$$\rho_o = 900$$
$$C_d = 0.61$$

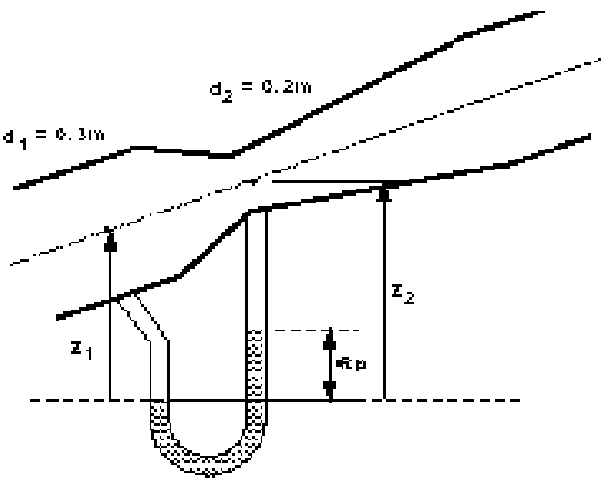
Apply Bernoulli,

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

Take atmospheric pressure as 0,

$$\frac{13780}{\rho_o g} + 0.61 = \frac{u_2^2}{2g}$$
$$u_2 = 6.53 m/s$$
$$Q = 0.61 \times 6.53 \times \pi \left(\frac{0.025}{2}\right)^2 = 0.00195 m^3/s$$

4-b)



What we know from the question:

$$\rho_g g = 19.62\ N/m^2$$
$$C_d = 0.96$$
$$d_1 = 0.3m$$
$$d_2 = 0.2m$$

Calculate Q.

$$u_1 = Q/0.0707 \qquad\qquad u_2 = Q/0.0314$$

For the manometer:

$$p_1 + \rho_g gz = p_2 + \rho_g g(z_2 - R_p) + \rho_w gR_p$$
$$p_1 - p_2 = 19.62(z_2 - z_1) + 587.423 \qquad <----(1)$$

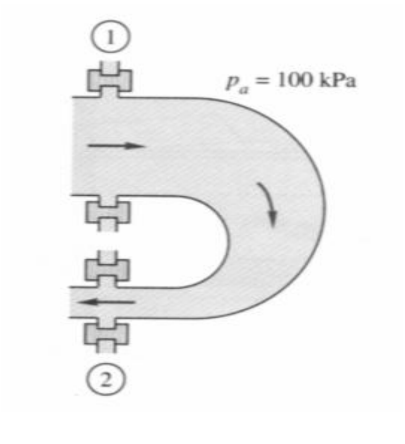
For the Venturimeter

$$\frac{p_1}{\rho_g g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho_g g} + \frac{u_2^2}{2g} + z_2$$
$$p_1 - p_2 = 19.62(z_2 - z_1) + 0.803u_2^2 \qquad <----(2)$$

Combining (1) and (2)

$$0.803u_2^2 = 587.423$$
$$u_{2ideal} = 27.047\ m/s$$
$$Q_{ideal} = 27.047 \times \pi\left(\frac{0.2}{2}\right)^2 = 0.85m^3/s$$
$$Q = C_d Q_{idea} = 0.96 \times 0.85 = 0.816m^3/s$$

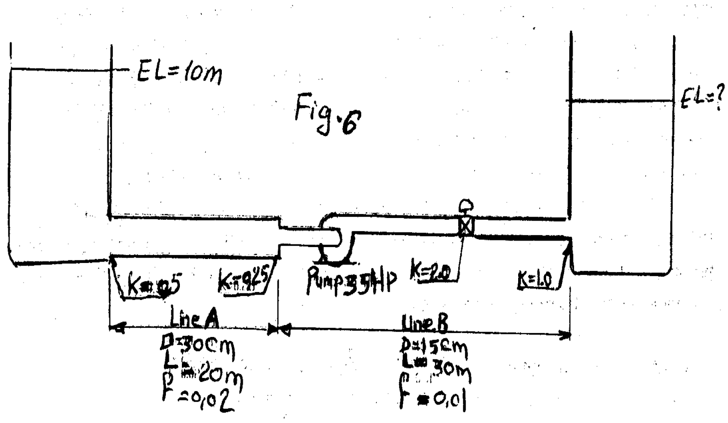
5-a)



$$\mathrm{Q} = \mathrm{A_1V_1} = A_2V_2 \quad \rightarrow \mathrm{V_2} = \frac{A_1}{A_2}V_1 = 21.48m/\sec$$

$$\dot{\mathrm{m}} = \rho \mathrm{A_1V_1} = 107\,kg/\sec \quad -F_{bolt} + P_{1g}A_1 + P_{2g}A_2 = \dot{m}(-21.48 - 2.2) \rightarrow F_{bolt} = 14.9287\,kN$$

5-b)



The velocity in pipe 1 u_1 = 2.122 m/sec , u_2 = 8.488 m/sec

H_{f1} = 0.02x20/0.3x(2.122)^2 /2g = 0.306 m

h_{in}= 0.267755 m

h_{cont}= 0.133877 m

H_{f2} = 0.01x30/0.15x(8.488)^2 /2g = 7.344 m

H_{out}= 3.672 m

H_{valve}= 7.344 m

Hp = 20m

El= 10.94 m