Benha University	**Department:Electrical Program**
Benha Faculty of Engineering	**Time:3 hours**
Date:5-2018	**Subject: Control Engineering**
Semester: 2	**Code:E1236**
Examiner:Dr.Shawky Arafah	**No. of Pages: 2**
Total Points:75	

Question ① (*20 marks*)

A- Write the most important **advantages and disadvantages** of the **open** loop and the **closed** loop control systems? (3)

B- Write the most important features of **a good** control system? (2)

C- A physical system consists of series **RLC** circuit as shown in Fig.1. The input is $v_i(t)$ and the output is the capacitor voltage $v_o(t)$. The system parameters are L=1 Henry, C=0.04 Farad, and R=6 Ohm.

i-Find a **mathematical model** and **Laplace model**? (2)

ii- Draw a unity feedback block diagram and find $V_o(S)/V_i(S)$? (3)

iii-Find the state space model using two state variables? (5)

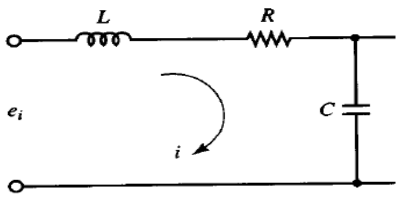

Fig.1

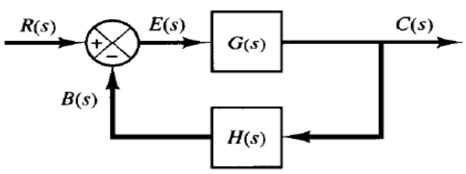

Fig.2

D- Find the closed loop transfer function of the system shown in Fig.3? (5)

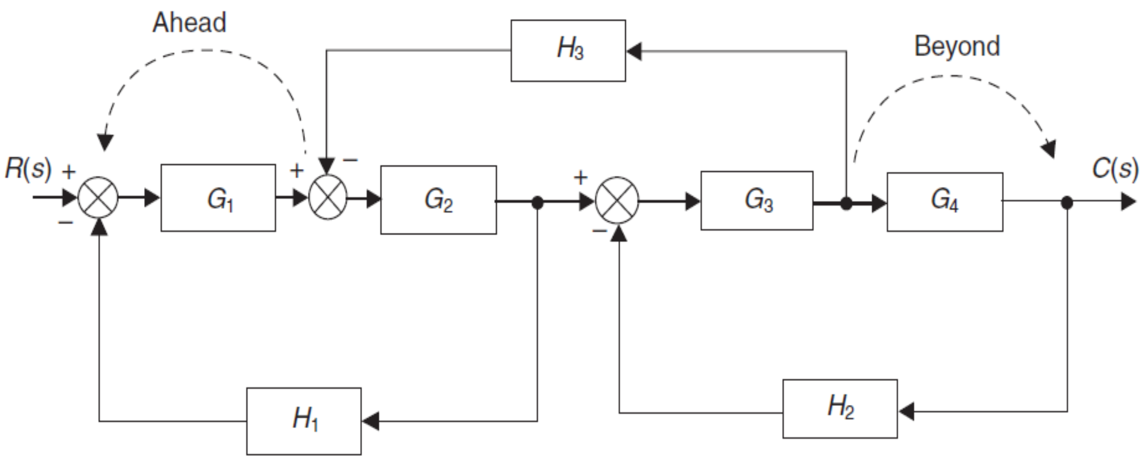

Fig.3

Question ② (***20 marks***)

A-Define: root locus- Bode diagram- polar plot- Nichols plot-gain margin- phase margin? (3)

B - Define: ω_n, ω_d, ω_r, ω_b, ω_g, ω_p, M_p, M_r, η? (3)

C- Consider a system shown in Fig. 2 has $H(s) = 1, \quad G(s) = \frac{K}{S(S+7)}$

i-Find the steady state static **error coefficients**? (3)

ii- Find the gain **K** such that the steady state ramp error =0.01? (1)

iii- Find and draw the **unit step response** as K=49? (5)

iv- Find the **frequency response** and $\mathbf{M_r}$ **and** $\boldsymbol{\omega_r}$ as K=49 and r(t)=sin ωt ? (5)

Question ③ (15 marks)

A- Define: input- output- disturbance- error- control- types of control system? (5)

B- Consider a unity feedback control system has (10)

$$G(s) = \frac{k(S+1)}{S(S+2)(S+5)} = \frac{k(S+1)}{S^3+7S^2+10S}$$

i-Sketch a **complete root locus** for positive values of **K**?

ii-Find **K** that makes the complex closed loop poles have a damping ratio =**0.6** and **find the closed loop poles** using **the plot**?

iv-Write short MATLAB program to solve i and ii?

Question ④ (20 marks)

Consider a control system shown in Fig.2 has an open loop TF=$G(s)H(s) = \frac{49}{S(S+7)}$

A-Prove that the gain margin=**infinite db at infinite rad/sec**. and the phase margin= **51.8 degrees at 5.5 rad/sec**.? (4)

B-Sketch the **polar plot**? (3)

C- Sketch the **Bode plot** and show gain margin and phase margin? (5)

D- Sketch the **Nichols plot?** (3)

E-Write short MATLAB program to solve a , b , C and D? (5)

Best Wishes for all,
Examiners

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Question ① (*20 marks*)

A- Write the most important **advantages and disadvantages** of the **open** loop and the **closed** loop control systems? (4)

Open loop control system

Advantages of open loop	disadvantages of open loop
1-simple construction	1-disturbances cause errors
2- ease of maintenance	2-changes in calibration cause errors
3-less expensive	3-recalibration is necessary
4-no stability problem	
5-convenient when output is hard to measured or economically not feasible	

Closed loop control system

Disadvantages of closed loop	advantages of closed loop
1-complex construction	1-disturbances do not cause errors
2- stability may be a problem	2- has less errors
3-more expensive	3-recalibration is not necessary
	4-the ability to adjust the response

B- Write the most important features of **a good** control system? (4)

Most important features of a good control system: are

1-simple construction and operation 2-fast response (speed) 3-less cost

4-very large accuracy (less error) 5-stable

C- A physical system consists of series **RLC** circuit as shown in Fig.1. The input is $v_i(t)$ and the output is the capacitor voltage $v_o(t)$.

i-Find a **mathematical model** and **Laplace model**? (2)

Mathematical model is:

$$\sum_1^n V_{loop} = 0,\quad \boldsymbol{v_i} - \boldsymbol{v_o} = Ri + L\frac{di}{dt} = 6i + \frac{di}{dt} \;, \boldsymbol{v_o} = \frac{1}{C}\int i(t)dt = 25\int i(t)dt$$

Laplace model with zero initial conditions is:

$$\boldsymbol{V_i}(S) - \boldsymbol{V_o}(S) = (R + SL)I(s) = (6 + S)I(s) \quad ,\boldsymbol{V_o}(S) = \frac{I(s)}{SC} = \frac{25\,I(s)}{S}$$

c- Draw a control block diagram and find $V_o(S)/V_i(S)$? (5)

the block diagram is shown in Fig 2 and

$$G(s) = \frac{1}{SC\,(R + SL)} = \frac{25}{S\,(6 + S)}, \qquad H(S) = 1,$$

$$TF = \frac{\boldsymbol{V_o}(S)}{\boldsymbol{V_i}(S)} = \frac{G(s)}{1 + G(S)H(S)} = \frac{{}^{1}/_{LC}}{S^2 + \frac{R}{L}S + {}^{1}/_{LC}} = \frac{25}{S^2 + 6S + 25}$$

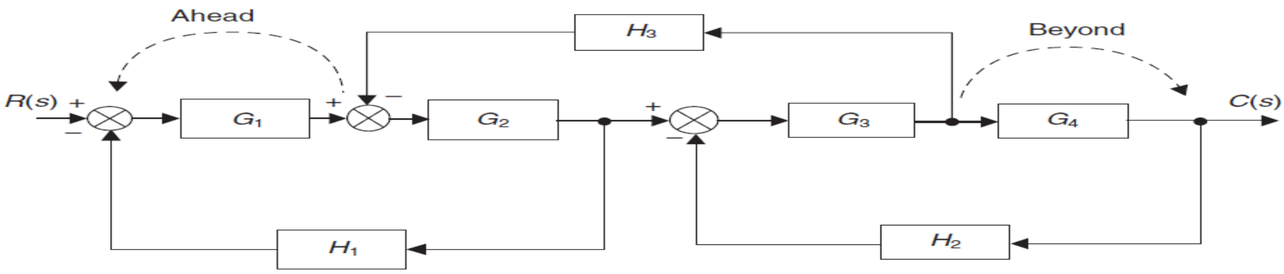
iii-Find the state space model using two state variables? (5)

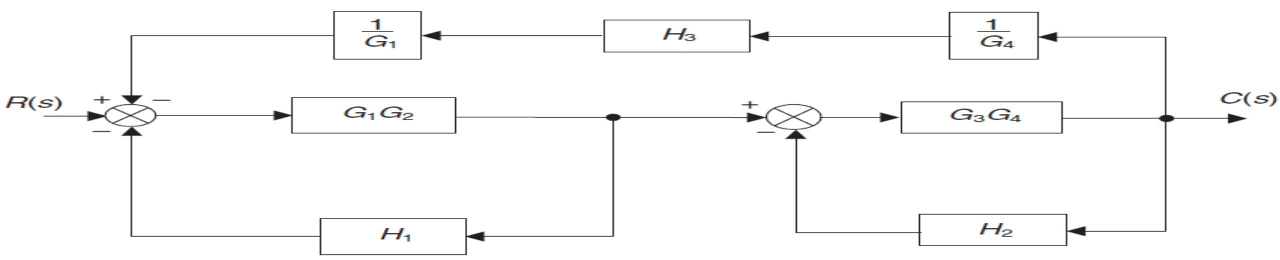
$$\boldsymbol{v_i} - \boldsymbol{v_o} - 6i = \frac{di}{dt}, \frac{d\boldsymbol{v_o}}{dt} = \frac{i(t)}{C} = 25i(t)$$

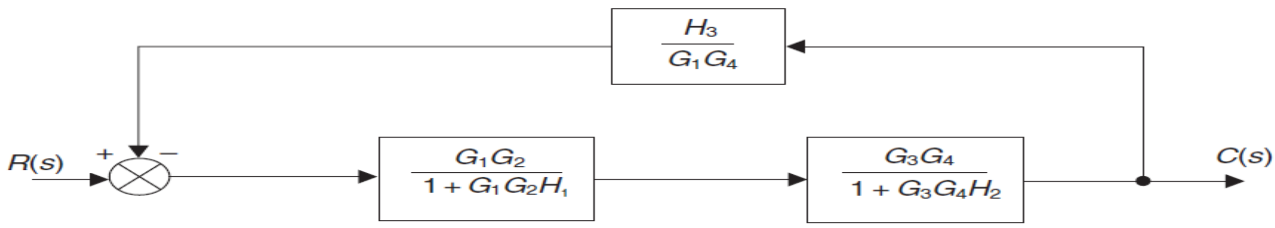
$$X^* = AX + BU, Y = CX + DU$$

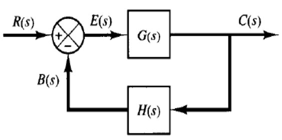
$$\begin{bmatrix} i^* \\ v_o^* \end{bmatrix}^* = \begin{bmatrix} -6 & -1 \\ 25 & 0 \end{bmatrix} \begin{bmatrix} i \\ v_o \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [V_i] \quad , [v_o] = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ v_o \end{bmatrix} + [0][V_i]$$

D- Find the closed loop transfer function of the system shown in Fig.3? (5)

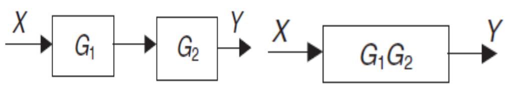








$$\frac{C}{R}(s) = \frac{G(s)}{1 + G(s)H(s)}$$



Question ② (20 marks)

A-Define: root locus- Bode diagram- polar plot- Nichols plot-gain margin- phase margin? (3)

The root-locus method is a very powerful graphical technique for investigating the effects of the variation of a system parameter on the location of the closed-loop poles. In most cases, the system parameter is the loop gain K, although the parameter

Bode diagram it consists of two parts on semi-log paper. The upper part is the plot of M magnitude in db against **ω** (log scale) and the lower part is the plot of the phase Φ in degrees against **ω** (log-scale).

polar plot (Nyquist): is the plot of the locus of the vector $\boldsymbol{M \angle \Phi}$ in the **Real-imaginary plane** where Φ in degrees as a straight line and determine M on this line as **ω** changes from zero to infinity

Nichols Plot: is the Plot **of M in db** on the vertical axis against **Φ in degrees** on the horizontal axis in the **X-Y plane**

-Gain margin G_m: it is reciprocal of the magnitude of the output frequency response at the **Phase crossover frequency ω_p**, G_m=1/[Real of G(j ω_p)H(j ω_p)]= $1/|G(j\,\omega p)H(j\,\omega p)|$ =K_c/K, G_M=20log G_m db

-Phase margin γ_m: it is the angle of the output frequency response at the **gain crossover** frequency plus 180 degrees. $\gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180$ deg.

B- Define: ω_n, ω_d, ω_r, ω_g, ω_p, M_p, M_r, η? (3)

-Natural frequency ω_n rad/sec**:** it is the natural frequency depends on the natural of the system parameters.

- Under damped natural frequency ω_d rad/sec**:** it is the under damped natural frequency depends on the damping coefficient **η** as it is less than one **η** <1.

-Resonant frequency ω_r rad/sec**:** it is the frequency at which the peak value of the output frequency response for a second order is equal to $\boldsymbol{\omega_r = \omega_n\sqrt{1-2\zeta^2}}$, **η** < 0.707.

As ζ approaches zero, M_r approaches infinity

$0 < \zeta \leq 0.707$, the resonant frequency ω_r is less than the damped natural frequency

-Gain crossover frequency ω_g: it is the frequency at which the magnitude of the output frequency response is equal to one or zero decibel.

$|G(j\,\omega g)H(j\,\omega g)| = 1$ or $|G(j\,\omega g)H(j\,\omega g)| = 0db$

-Phase crossover frequency ω_p: it is the frequency at which the phase of the output frequency response is equal to (-180) degrees.

Imag. [G(j ω_p)H(j ω_p)]=0 or $\angle G(j\,\omega p)H(j\,\omega p) = -180$deg.

-Maximum resonant magnitude M_r: it is the peak value of the output frequency response for a second order system $\boldsymbol{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad M_r = |G(j\omega)|_{\max} = |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

-damping coefficient η it depends on the natural of the system parameters. For second order system

Values of η	System stability	Step-response
0> η	System is unstable	undefined
η =0	System is critically stable	oscillatory
0< η <1	System is stable	Under-damped
0< η =1	System is stable	Critically damped
0<η >1	System is stable	Over damped

C- Consider a system shown in Fig. 2 has $H(s) = 1, \quad G(s) = \frac{K}{S(S+7)}$

i-Find the steady state static **error coefficients**? (3)

as $K_p = \lim_0 G(S) = \lim_0 \frac{K}{S(S+7)} = \frac{K}{(0)(0+7)} = \infty$

$K_v = \lim_0 SG(S) = \lim_0 \frac{K}{(S+7)} = \frac{K}{(0+7)} = 0.14K,\ K_a = \lim_0 S^2G(S) = \lim_0 \frac{S\,K}{(S+7)} = 0$

ii- Find the gain **K** such that the steady state ramp error =0.01? (1)

$e_{ss}(t) = \frac{1}{K_v} = \frac{1}{0.14K} = 0.01,\ K = 714,$ Routh test as K=714, system is stable for positive K

c-Find and draw the **unit step response** as K=49?

$$\frac{C(S)}{R(S)} = \frac{G(s)}{1 + G(S)H(S)} = \frac{\omega_n{}^2}{S^2 + 2\eta\omega_n S + \omega_n{}^2} = \frac{49}{S^2 + 7S + 49}, \omega_n = 7\,\text{rad/sec.},\ \ \eta = 0.5$$

The step response is under damped Step response of a second order system R(S)=1/s

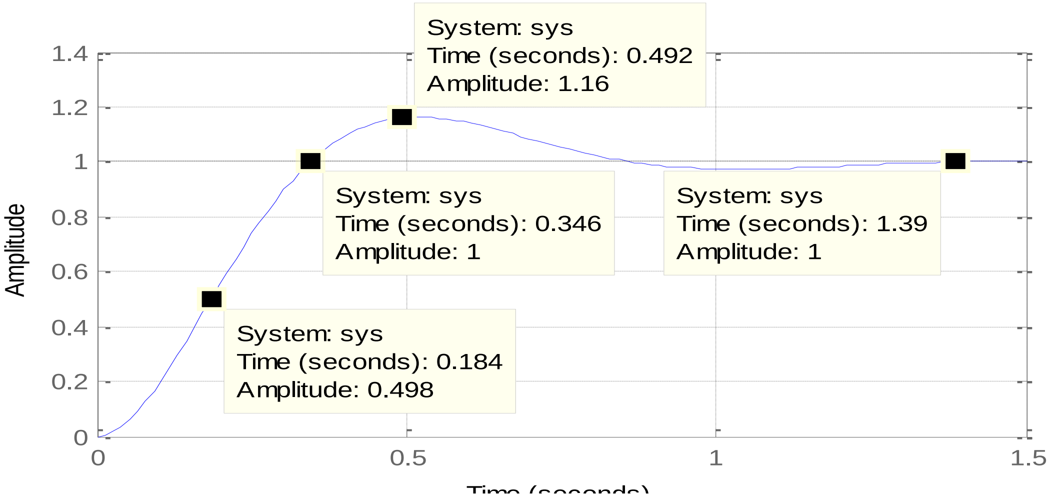
C(S) =closed loop T.F(S)*R(S) $= \frac{\omega_n{}^2 R(S)}{S(S^2+2\eta\omega_n S+\omega_n{}^2)} = \frac{49}{S(S^2+7S+49)} = \frac{a}{S} + \frac{bs+d}{(S^2+7S+49)}$ *partial fraction* ,

C(t)= inverse Laplace of the product of closed loop t.f.(S) and R(S)=1/s with zero initial conditions C(t)= L^{-1}[(C(S)]=L^{-1}[closed loop t.f.(S)*R(S)] with zero initial conditions]

$$\eta = 0.5, \omega_n = 7 rad/sec.\ \omega_d = \omega_n\sqrt{1-\eta^2} = 6.1\, rad/sec\ , cos^{-1}0.5 = pi/3$$

$$C(t) = 1 - \frac{e^{-\eta\ \omega_n t}}{\sqrt{1-\eta^2}}\sin(\omega_d t + cos^{-1}\eta) \quad = 1 - 1.155e^{-3t}\sin(6.1t + pi/3)$$

$$M_{p=}e^{\frac{-\eta\pi}{\sqrt{1-\eta^2}}} = 0.163, t_r = \frac{\pi - cos^{-1}\eta}{\omega_d} = \frac{\pi - \frac{pi}{3}}{6.1} = 0.34sec, t_p = \frac{\pi}{\omega_d} = 0.51sec., t_s = 4T = \frac{4}{\eta\omega_n} = 1.14sec.$$



iv- Find the **frequency response** and $\mathbf{M_r}$ **and** $\boldsymbol{\omega_r}$ as K=49 and r(t)=sin ωt ? (5)

$$\frac{C(S)}{R(S)} = \frac{G(s)}{1+G(S)H(S)} = \frac{\omega_n{}^2}{S^2+2\eta\omega_n S+\omega_n{}^2} = \frac{49}{S^2+7S+49}$$

$$\omega_n = 7\,\text{rad/sec.},\ \ \eta = 0.5, M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{1}{2(0.5)\sqrt{1-(0.5)^2}} = 1.155$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} = 7\sqrt{1-2(0.5)^2} = 4.95\ \text{rad/sec.}$$

the frequency response as r(t)=sinωt . Steps to find frequency Response:

1- the closed loop transfer function =**T(s)=C(S)/R(S)** =

$$\frac{C(S)}{R(S)} = \frac{G(s)}{1+G(S)H(S)} = \frac{\omega_n{}^2}{S^2+2\eta\omega_n S+\omega_n{}^2} = \frac{49}{S^2+7S+49}$$

2-the closed loop frequency transfer function =**T (jω)=C(jω)/R(jω)** $= \frac{49}{(j\omega)^2+7(j\omega)+49} =$

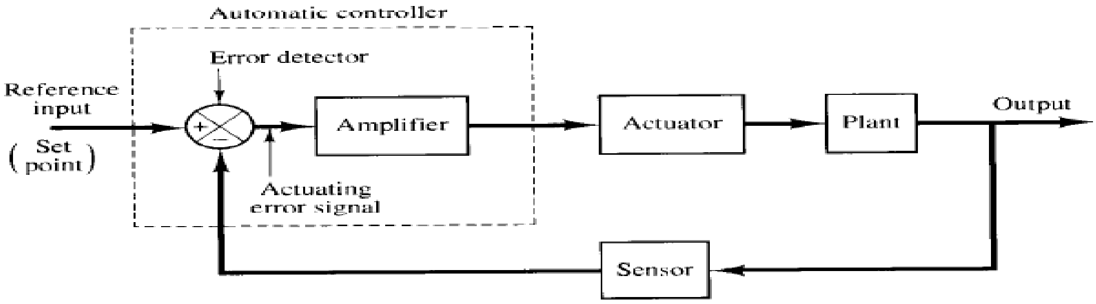
M∟Φ=Re+j imag, $\boldsymbol{M} = \frac{49}{\sqrt{(49-\omega^2)^2+49\omega^2}}$, $\boldsymbol{\Phi = -\tan^{-1}[7\omega/(49-\omega^2)]}$

3-As the input =$r(t) = sin\omega t$ then the response = $C(t) = Msin(\omega t + \Phi)$

$$= \frac{49}{\sqrt{(49-\omega^2)^2+49\omega^2}} \sin[\omega t - \tan^{-1}[7\omega/(49-\omega^2)]]$$

<u>Question ③ (15 marks)</u>

A- Define: input- output- disturbance- error- control- types of control system? (5)



Control: it means:

1-applying the desired value (reference or control input) to the system

2-measuring the value of the output (controlled variable) of the system

3-determining the defference between the input and the output (error)

4-applying the error (manipulated variable) to the controller

5-choose the controller such that the error is correctted or limited

4-Iutput=set point=reference=testing signal=forcing signal=desired value = r(t)**:**

it is a control variable input (signal) to a system and may be:

1-impulse 2-step 3-ramp 4-parabolc 5-exponential 6-sinusoidal

5- Controlled variable=output=response=measured value=actual output =C(t): it is the ultimate output of the process; the actual parameter of the process that is being controlled. Or it means the quantity or condition of the output of the system or the measured value of the system which be controlled

6- The (error) manipulated variable: it means the quantity or condition that is varied by the controller so as to affect the value of the controlled variable. **Error**: it is the difference between the input and the output for a unity feedback control systems only. Or it is the difference between the input and the feedback signal for a non-unity feedback control systems. It has two types static and dynamic.

1-static error depends on two; the input and system parameters (T.F.)

2-dynamic error depends on three; the input and derivatives of the input and system parameters (T.F.).

7- **Disturbance**: it is a signal tends to adversely affect the value of the output of a system. It is may be internal (generated within the system) or external (generated outside the system)=d(t).

Types of control systems:

A-depend on control loop are: 1-open loop 2-closed loop

B-depend on the control algorithm are:

1-Regulator system it automatically maintains a parameters at (or near) a specified value such as home heating system.

2-Follow-up system it causes an output to follow a set path that has been specified in advance such as industrial robot moving parts from place to place.

3-Event control system it controls a sequential series of event such as a washing machine cycling through a series of programmed steps.

C-depend on DE are: 1-linear 2-nonlinear

linear system: it is a system in which the response to several inputs can be calculated by treating one input at a time and adding the results i.e. can be apply the superposition theorem (the response produced by the simultaneous application of two different forcing functions is the sum of the two individaul resposes). Its advantages are: 1-can solve analytically, 2-once solved the solution is valid,

3-superposition can applied, 4-response characteristics are consistent

D-depend on the independent variables of the DE are: 1-time-varient 2-time-invarient

Time invariant system: it is a system which model has differential equations in which the coefficients and the independent variables are constants.

E-depend on the order of the characteristic equation are: 1-first order, 2 second order, 3- higher order

Question ③ (15 marks)

B-Consider a unity feedback control system has (10)

$$G(s) = \frac{k(S+1)}{S(S+2)(S+5)} = \frac{k(S+1)}{S^3+7S^2+10S}$$

i-Sketch a **complete root locus** for positive values of **K**?

ii-Find **K** that makes the complex closed loop poles have a damping ratio =**0.6** and **find the closed loop poles** using **the plot**?

iv-Write short MATLAB program to solve i and ii?

a-Root locus:

1-the root locus is symmetrical about the real axis in the S-plane

2-the open loop $G(s)H(s) = \frac{k(S+1)}{S(S+2)(S+5)} = \frac{k(S+1)}{S^3+7S^2+10S}$

3-the root locus starts at the poles and ends at the zeros or infinity

4-number of root loci= n=number of poles of the open loop TF =3 atS= [0,-2,-5]

5-number of zeros= m=1 at S=-1

6-number of asymptotes = n-m=3-1=2

8-center of gravity = $A = \frac{\sum \text{poles} - \sum \text{zoles}}{\text{n}-\text{m}} = \frac{-2-5-1}{2} = -3$ point of intersection of asymptotes with real axis, 9-angles of asymptotes are = $\Theta = \frac{\pm 180(2R+1)}{n-m} = \pm 180/2 = \pm 90$ degrees

10- Points of crossing the imaginary axis as Routh test

Charct.equa=1+G(S)H(S)=0= $S^3+7S^2+(10+K)S+K=0$

S^3	1	10+K	K≥0, [70+6K]/ 7≥0 then -11.7≤K,0≤K, Kc=all positive values No intersection with jw-axis
S^2	7	K	
S	[70+6K]/ 7		
S^0	K		

11- there is one break point (break away) at

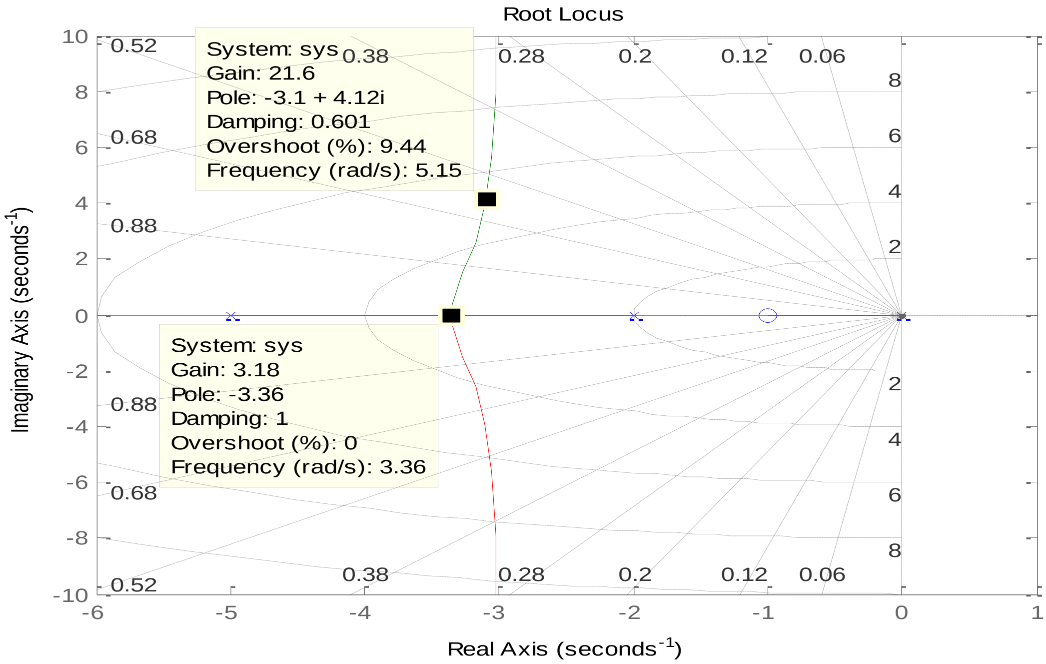
$$-\frac{dK}{dS} = 0 = \frac{d}{dS}\left[\frac{1}{G(S)H(S)}\right] = \frac{d}{dS}\left[\frac{\text{k}(\text{S}+1)}{\text{S}^3+7\text{S}^2+10\text{S}}\right] = S^3 + 5\text{S}^2 + 7\text{s} + 5 = 0$$

roots([1 5 7 5]), ans = -3.3593, -0.8203 + 0.9030i, -0.8203 - 0.9030i

12- There is two break angles [±180(2R+1)/r=[±180(2R+1)/2=[±90]

13-the angle of departures (complex poles) =no, 14- the angle of arrival (complex zeros)=no

15-sketch the root loci as



16- the damping factor or coefficient ζ is straight line with slope $\Theta = \cos^{-1}\zeta$

with respect to the negative real axis in the S-plane. $\Theta = \cos^{-1} 0.6 =$ deg. at the test point (intersection point)

S_d=-3±j4, angle condition $= \sum_{n=1}^{n=3}[\Theta_{zeros} - \Theta_{poles}] = \pm 180(2R+1) = 90 + 54 + 36 = 180$ deg

$$\text{magnitude condition} = \text{products of } \frac{\|\text{poles}\|}{\|\text{zeros}\|} = K = \; * \;\; * * / * = 21.5$$

$$\sum_{n=1}^{n=3} \textit{open loop poles} = \sum_{n=1}^{n=3} \textit{closed loop poles} = \text{constant as } n - m \geq 2$$

$$\sum_{n=1}^{n=3} \text{open loop poles} = -0 - 2 - 5 = -7 = \sum_{n=1}^{n=3} \text{closed loop poles} = (-3.+j4, -3.-j4, p)$$

$$\text{then } p = -1 \text{ i.e. closed loop poles are } [-3.\pm j4, -1\,]$$

Prog. >>n=[1 1];d=[1 7 10 0]; rlocus(n,d), grid

__Question__ ④ (***__20 marks__***)

Consider a control system shown in Fig.1 has an open loop TF=$G(s)H(s) = \frac{49}{S(S+7)}$

A-Prove that the gain margin=**infinite db at infinite rad/sec**.

and the phase margin= **51.8 degrees at 5.5 rad/sec**.? (4)

1- the open loop TF=G(s) H(s)= **G(S) H(S)**

$$G(s)H(s) = \frac{49}{S(S+7)} = \frac{49}{S^2+7S}$$

2- Find the freq.open loop TF=

$$G(j\omega)H(j\omega) = \frac{49}{S^2+7S} = \frac{49}{j\omega\,(j\omega+7)} = \frac{49}{-\omega^2+7j\omega} = Me^{j\Phi} = M\llcorner\Phi = \text{Re}+j\,\text{imag}$$

$$M = \frac{49}{\omega\sqrt{\omega^2+49}},\ \Phi = -90-\tan^{-1}(\frac{\omega}{7}),\ Real = \frac{-49}{\omega^2+49},\ \text{Imag} = \frac{-343}{\omega(\omega^2+49)}$$

$$M = \frac{49}{\omega\sqrt{\omega^2+49}} = \frac{49}{5.5\sqrt{5.5^2+49}} = 1,\ \text{then } \omega_g = 5.5 rad/sec.$$

$\Phi = -180$ deg. then $\omega_p = infinite\ rad/sec.$

$$M = \frac{36}{\omega\sqrt{\omega^2+36}} = 0\ at\ infinite\frac{rad}{sec}.,\text{then } G_M = 20log\frac{1}{0} = infinitedb$$

$\Phi = -90-\tan^{-1}(4.72/6) = -90-38.2 = -128.2,\qquad \gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180\text{ deg.} = 180-128.2 = 51.8deg.$

3- Find the table

ω	0	0.1	1	4.72	5	10	100	∞
Φ								
M								
20logM								
Real **G(jω)H(jω)**								
Imag **G(jω)H(jω)**								

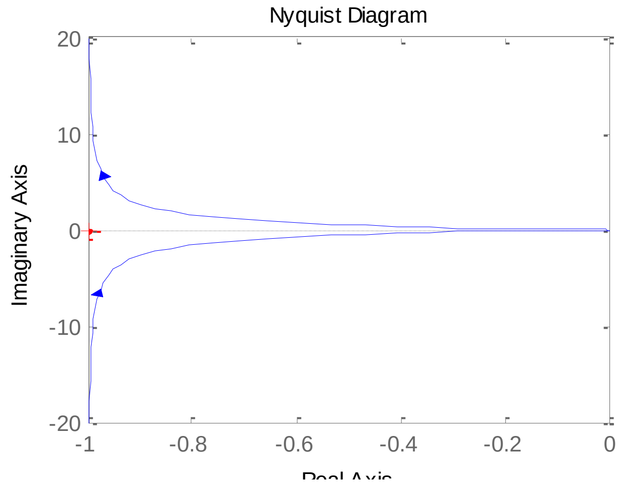
B-Sketch the **polar plot**? (3)

C- Sketch the **Bode plot** and show gain margin and phase margin? (5)

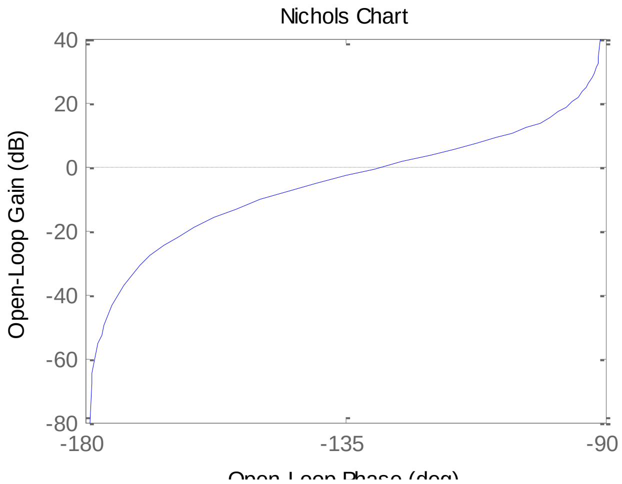
D- Sketch the **Nichols plot?** (3)

E-Write short MATLAB program to solve a , b , C and D? (5)

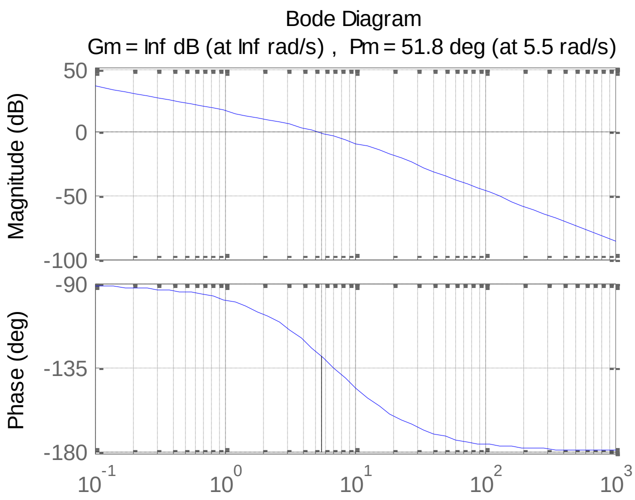
Prog. >>n=[49]; d=[1 7 0]; >> nyquist(n,d) >> margin(n,d) >> nichols(n,d)



Nyquist Diagram
Imaginary Axis
20
10
0
-10
-20
-1
-0.8
-0.6
-0.4
-0.2
0



Nichols Chart
Open-Loop Gain (dB)
40
20
0
-20
-40
-60
-80
-180
-135
-90



Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 51.8 deg (at 5.5 rad/s)
Magnitude (dB)
50
0
-50
-100
Phase (deg)
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}