Benha University
Benha Faculty of Engineering
Date: 30/05/2018
Semester: Two
Examiner: Dr. Hussein Shehata
Total Points: 90 marks

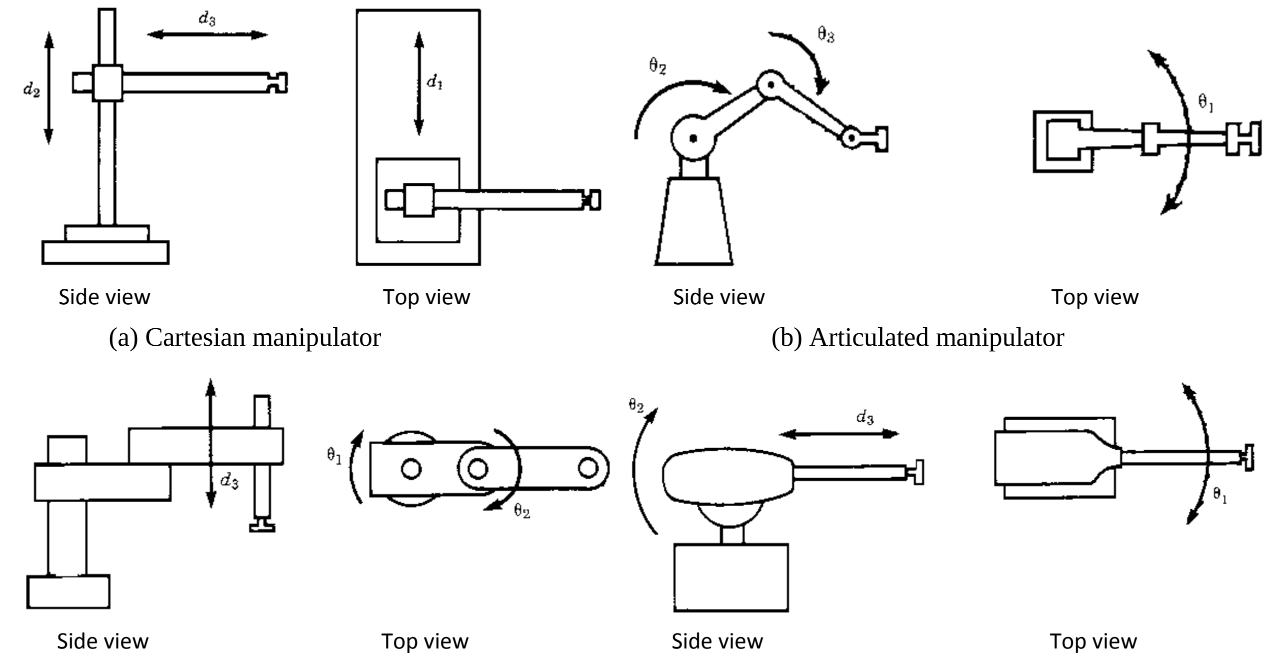
Department: Mechanical Eng.
Program: Mechatronics
Time: 3 hours.
Subject: Robotics
Code: (M1596)
No. of Pages: 2

Model Answer

Questions ① and ② are mandatory for All Students.

Question ① (20 marks)

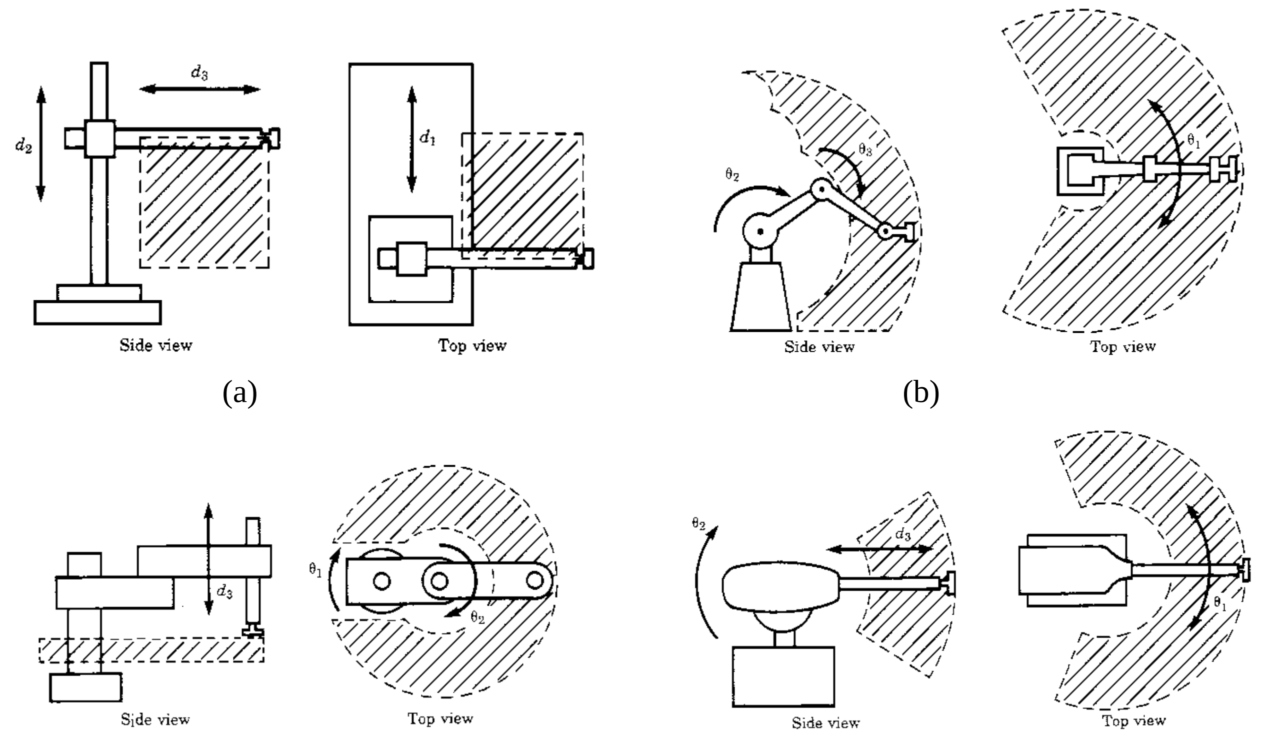
Fig. Q.1 depicts some schematic drawing for **Cartesian, Articulated, SCARA, and Spherical** manipulators, respectively. **Draw a freehand sketch** for the estimated **workspace**.



(a) Cartesian manipulator (b) Articulated manipulator (c) SCARA manipulator (d) Spherical manipulator

Fig. Q1: Schematic drawing of serial manipulators (05 each)

Solution ①



(a) (b) (c) (d)

Question ② (20 marks)

a) Give your own comment with an example to validate that the rotation sequence ***XYZ fixed frame*** is equivalent to ***ZYX Euler***. (05)

b) **Fig. Q.2** shows a two-link manipulator with rotational joints. Calculate the **velocity** and **Jacobian** of the tip of the arm as a function of joint rates. Give the answer in two forms, in terms of frame **{3}** and also in terms of frame **{0}**. (10)

c) General mechanisms, sometimes, have certain configurations, called **"isotropic points, "***where the columns of the Jacobian become **orthogonal** and of **equal magnitude**"*. Referring to part **(b)**, find out if any isotropic points exist. **Hint: Is there a requirement on L1 and L2?** (05)

$$ {}^{3}J(\theta) = \begin{bmatrix} l_1 S_2 & 0 \\ l_1 c_2 + l_2 & L_2 \end{bmatrix} $$

$$ \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{(i-1)} \\ \sin\theta_i \cos\alpha_{(i-1)} & \cos\theta_i \cos\alpha_{(i-1)} & -\sin\alpha_{(i-1)} & -\sin\alpha_{(i-1)} d_i \\ \sin\theta_i \sin\alpha_{(i-1)} & \cos\theta_i \sin\alpha_{(i-1)} & \cos\alpha_{(i-1)} & \cos\alpha_{(i-1)} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

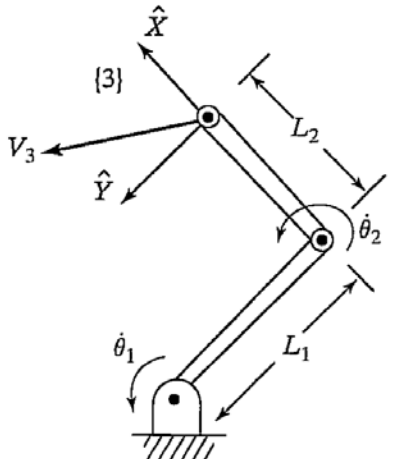


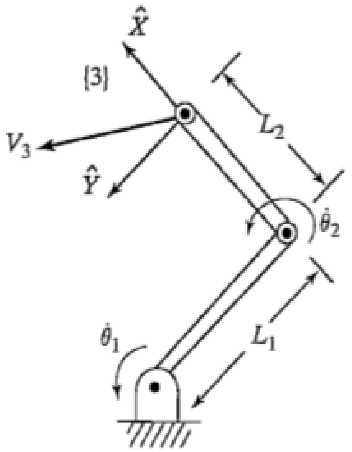
Fig. Q.2: A two-link manipulator

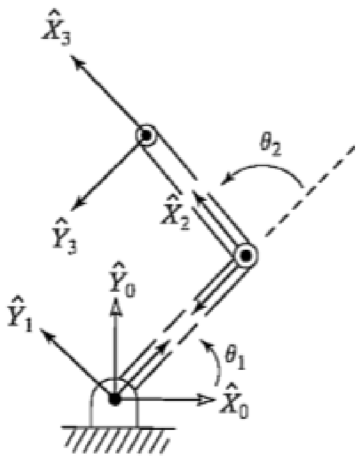
Solution ②

a)

XYZ fixed frame	***ZYX Euler***
Order:	Order:
rotation around X by alpha	rotation around Z by alpha
Followed by rotation around Y by Beta	Followed by rotation around new Y by Beta
Followed by rotation around Z by Gamma	Followed by rotation around new X by Gamma
It is called Extrinsic	It is called Intrinsic
$[{}^{A}P] = R_Z R_Y R_X [{}^{B}P]$	$[{}^{A}P] = R_Z R_Y R_X [{}^{B}P]$

b)





i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	l_1	0	θ_2
3	0	l_2	0	0

Table 1: D-H Parameters

$$
{}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^2_3T = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

$$
{}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}. \quad {}^{i+1}v_{i+1} = {}^{i+1}_{i}R({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1}).
$$

$$
{}^1\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix},
$$

$$
{}^1v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},
$$

$$
{}^2\omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix},
$$

$$
{}^2v_2 = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_1\dot{\theta}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} l_1 s_2 \dot{\theta}_1 \\ l_1 c_2 \dot{\theta}_1 \\ 0 \end{bmatrix}
$$

$$
{}^3\omega_3 = {}^2\omega_2,
$$

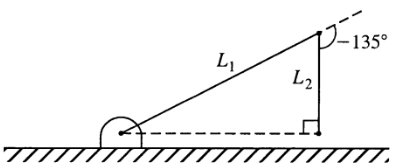
$$
{}^3v_3 = \begin{bmatrix} l_1 s_2 \dot{\theta}_1 \\ l_1 c_2 \dot{\theta}_1 + l_2(\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix}. \qquad {}^3J(\Theta) = \begin{bmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{bmatrix},
$$

$$
{}^0_3R = {}^0_1R \;\; {}^1_2R \;\; {}^2_3R = \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}.
$$

$$
{}^0v_3 = \begin{bmatrix} -l_1 s_1 \dot{\theta}_1 - l_2 s_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ l_1 c_1 \dot{\theta}_1 + l_2 c_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix}. \qquad {}^0J(\Theta) = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}.
$$

c)

The Jacobian of this 2-link is:



$$
{}^3J(\underline{\theta}) = \begin{bmatrix} L_1 S_2 & 0 \\ L_1 C_2 + L_2 & L_2 \end{bmatrix}
$$

An isotropic point exists if

$$
{}^3J = \begin{bmatrix} L_2 & 0 \\ 0 & L_2 \end{bmatrix} \text{ so,}
$$

$$
L_1 S_2 = L_2
$$

$$
L_1 C_2 + L_2 = 0
$$

or, $S_2 = \dfrac{L_2}{L_1} C_2 = \dfrac{-L_2}{L_1}$

Now $S_2^2 + C_2^2 = 1$,

so $\left(\frac{L_2}{L_1}\right)^2 + \left(\frac{-L_2}{L_1}\right)^2 = 1$

or $L_1^2 = 2L_2^2 \rightarrow L_1 = \sqrt{2}L_2$

Under this condition $S_2 = \frac{1}{\sqrt{2}} = \pm.707$

and $C_2 = -.707$

$\therefore$ An isotropic point exists if $L_1 = \sqrt{2}L_2$ and in that case it exists when $\theta_2 = \pm 135°$

In this configuration, the manipulator looks momentarily like a Cartesian manipulator.

For the remaining questions, all students are required to solve 2 QUESTIONS ONLY.

Question ③ (25 marks)

a) What might be meant by the statement: "An *n*-DOF manipulator at a singularity can be treated as a redundant manipulator in a space of dimensionality (*n* – 1)". (10)

b) For the three-link manipulator shown in **Fig. Q.3**, give a set of joint angles for which the manipulator is at a **workspace-boundary singularity** and another set of angles for which the manipulator is at a **workspace-interior singularity**. (15)

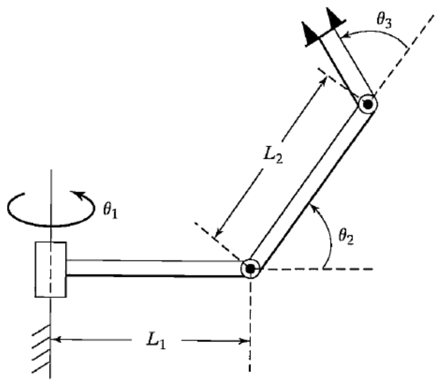
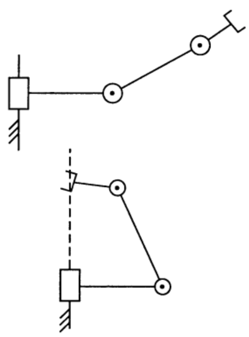


Fig. Q.3: The 3R non-planar arm manipulator

Solution ③

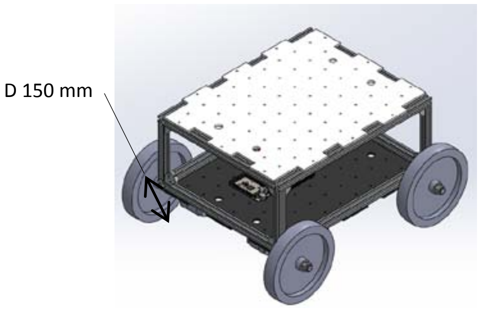
a) The statement means that "at a singularity an *n*-DOF robot only has N-1 DOF remaining. Hence, it can move freely in some N-1 dimensional subset. However, it is still true that it has *n* joints. Therefore, we have a device which has one more joint than the dimensionality of the space it is described in – and that is what a redundant manipulator is.



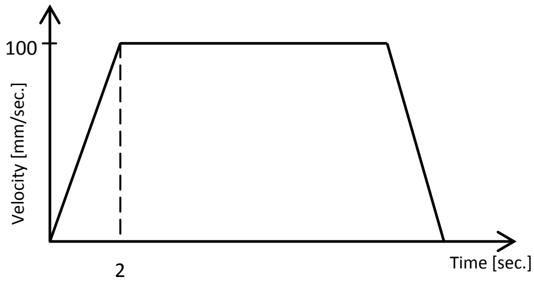
b) **Workspace boundary:** any angle set: $\{\theta_1, \theta_2, 0\}$
Workspace interior: any angle set such that:
$L_1 + L_2C_2 + L_3C_{23} = 0$…….(θ_1 is arbitrary)

Question ④ (25 marks)

When building a mobile robot, selecting the drive motors is one of the most important decisions you will make. **Assume the 4 Wheeled Drive Mobile Robot (4WD-MR)** as depicted in ***Fig. Q.4,*** with a total mass of **75 kg** is travelling with a **trapezoidal motion profile.** You are required to make a complete motor sizing for this application, considering the **worst case**. *Assume any data that you need.*



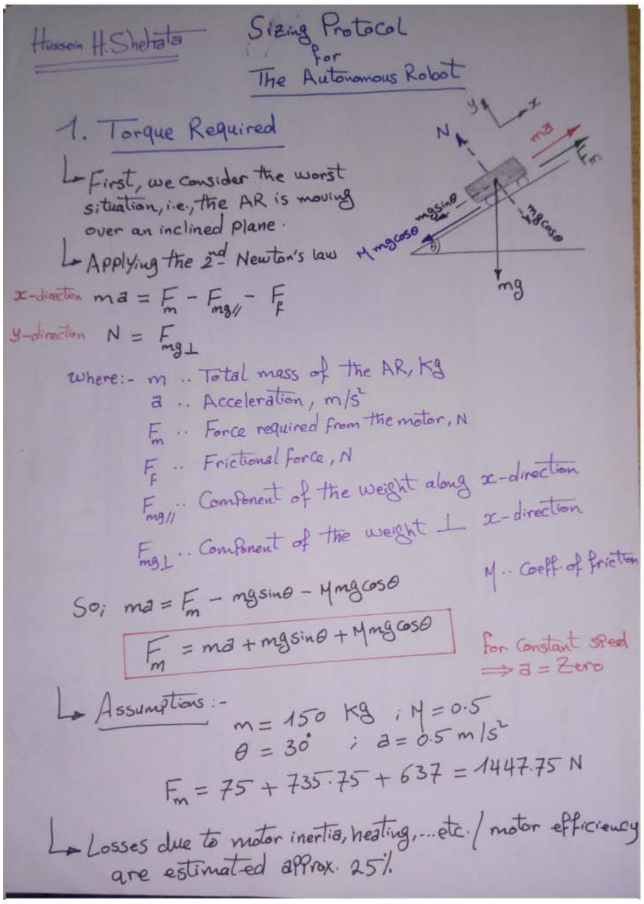
(a) 4WD-MR



(b) Trapezoidal motion profile

Fig. Q.4: 4WD-MR

Solution ④



Hussein H. Shehata

Sizing Protocol for The Autonomous Robot

1. Torque Required

↳ First, we consider the worst situation, i.e., the AR is moving over an inclined plane.

↳ Applying the 2nd Newton's law

x-direction $ma = F_m - F_{mg//} - F_F$

y-direction $N = F_{mg\perp}$

where:-
- m .. Total mass of the AR, Kg
- a .. Acceleration, m/s^2
- F_m .. Force required from the motor, N
- F_F .. Frictional force, N
- $F_{mg//}$ .. Component of the weight along x-direction
- $F_{mg\perp}$ .. Component of the weight ⊥ x-direction
- μ .. Coeff. of friction

So; $ma = F_m - mg\sin\theta - \mu mg\cos\theta$

$F_m = ma + mg\sin\theta + \mu mg\cos\theta$

For constant speed ⟹ a = Zero

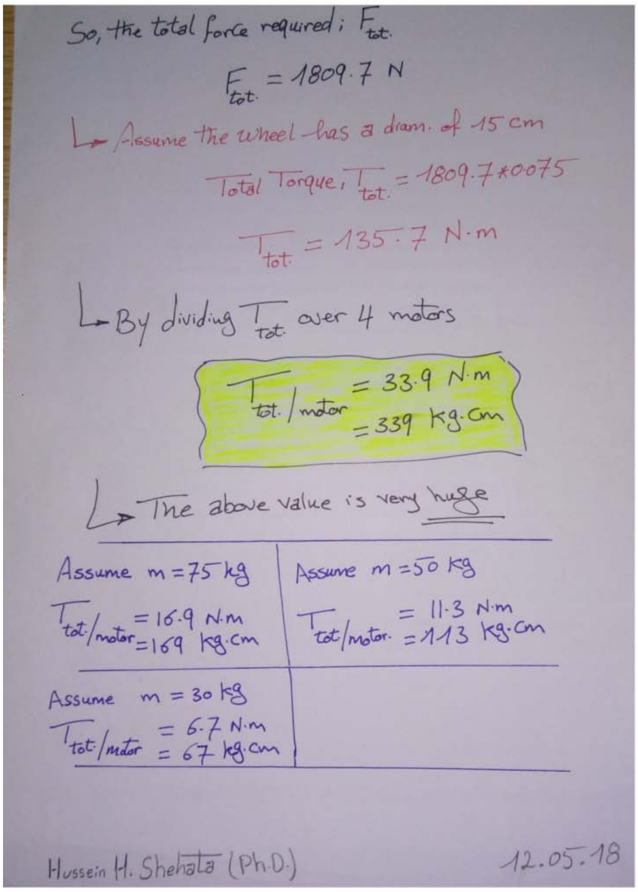
↳ Assumptions:-

$m = 150$ kg ; $\mu = 0.5$

$\theta = 30°$; $a = 0.5\ m/s^2$

$F_m = 75 + 735.75 + 637 = 1447.75$ N

↳ Losses due to motor inertia, heating, ...etc. / motor efficiency are estimated approx. 25%



So, the total force required; $F_{tot.}$

$F_{tot.} = 1809.7$ N

↳ Assume the wheel has a diam. of 15 cm

Total Torque, $T_{tot.} = 1809.7 * 0.075$

$T_{tot.} = 135.7$ N·m

↳ By dividing T_{tot} over 4 motors

$T_{tot./motor} = 33.9$ N·m $= 339$ kg·cm

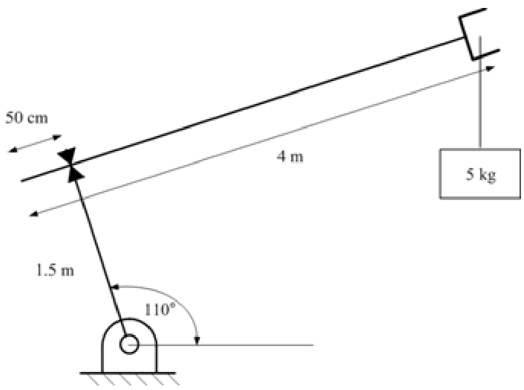
↳ The above value is very huge

Assume m = 75 kg $T_{tot/motor}$ = 16.9 N·m = 169 kg·cm	Assume m = 50 kg $T_{tot/motor}$ = 11.3 N·m = 113 kg·cm
Assume m = 30 kg $T_{tot/motor}$ = 6.7 N·m = 67 kg·cm	

Hussein H. Shehata (Ph.D.) 12.05.18

Question ⑤ (25 marks)

Fig. Q.5 depicts a schematic diagram of the 2-DOF long-reach robot arm. The first link weighs **50 kg** and the second link **10 kg**. Their total lengths are **1.5 m** and **4 m**, respectively. At the depicted posture, where the first link is rotated by **110°** and the second link is traveled by **3.5 m** forward, determine the required torque and force of both actuators to maintain the configuration. For simplicity, let the C.G. of the links be at their mid-point.



Best Wishes for all,
Dr. Hussein Shehata

Fig. Q.5: Schematic diagram of the 2-DOF long-reach robot arm

Solution Guide ⑤

6.2.1 Force and Moment Balance of a Link

In a serial manipulator, each link is connected to one or two other links by various joints. Figure 6.2 depicts the forces and moments acting on a typical link i that is connected to link $i-1$ by joint i and to link $i+1$ by joint $i+1$. The forces acting on link $i+1$ by link i in joint $(i+1)$ can be reduced to a resultant force $\mathbf{f}_{i+1,i}$ and a resultant moment $\mathbf{n}_{i+1,i}$ about the origin O_i of the (x_i, y_i, z_i) link coordinate frame. Similarly, the forces acting on link i by link $i-1$ in the ith joint can be reduced to a resultant force $\mathbf{f}_{i,i-1}$ and a moment $\mathbf{n}_{i,i-1}$ about the origin O_{i-1} of the $(x_{i-1}, y_{i-1}, z_{i-1})$ link coordinate frame. The following notations are defined:

- $\mathbf{f}_{i+1,i}$: resulting force exerted on link $i+1$ by link i at O_i, $\mathbf{f}_{i,i+1} = -\mathbf{f}_{i+1,i}$.
- $\mathbf{g}$: acceleration of gravity.
- m_i: mass of link i.
- $\mathbf{n}_{i+1,i}$: resulting moment exerted on link $i+1$ by link i, about point O_i, $\mathbf{n}_{i,i+1} = -\mathbf{n}_{i+1,i}$.

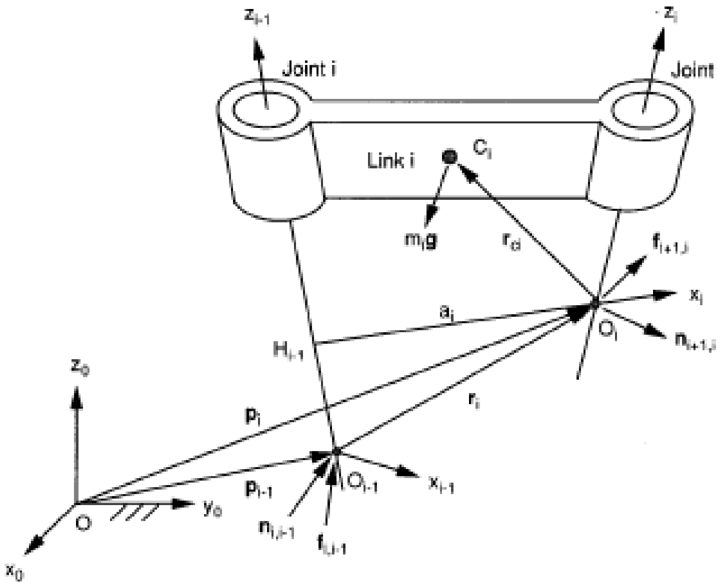


FIGURE 6.2 Forces and moments acting on link i.

- $\mathbf{r}_{ci}$: position vector of the center of mass of link i relative to the ith link frame (i.e., $\mathbf{r}_{ci} = \overline{O_iC_i}$).
- $\mathbf{r}_i$: position vector of O_i with respect to the $(i-1)$th link frame (i.e., $\mathbf{r}_i = \overline{O_{i-1}O_i}$).

Example 6.2.1 *Statics of a Planar 3-DOF Manipulator* Figure 6.3 shows the planar $3R$ manipulator studied in Chapters 2 and 4. A coordinate system with all the z-axes pointing out of the paper is defined for each link according to the D-H convention. Let the end-effector output force and moment be given by $\mathbf{f}_{4,3} = [f_x, f_y, 0]^T$ and $\mathbf{n}_{4,3} = [0, 0, n_z]^T$, respectively. Also let the acceleration of gravity, $\mathbf{g}$, be pointing along the negative y_0-direction and the center of mass be located at the midpoint of each link. We wish to find the joint reaction forces and moments.

The D-H parameters and transformation matrices are given in Table 2.1 and Eqs. (2.6) through (2.8). The vectors ${}^i\mathbf{r}_i$ and ${}^i\mathbf{r}_{ci}$ are

$$ {}^i\mathbf{r}_i = \begin{bmatrix} a_i \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad {}^i\mathbf{r}_{ci} = \begin{bmatrix} -a_i/2 \\ 0 \\ 0 \end{bmatrix}. \tag{6.18} $$

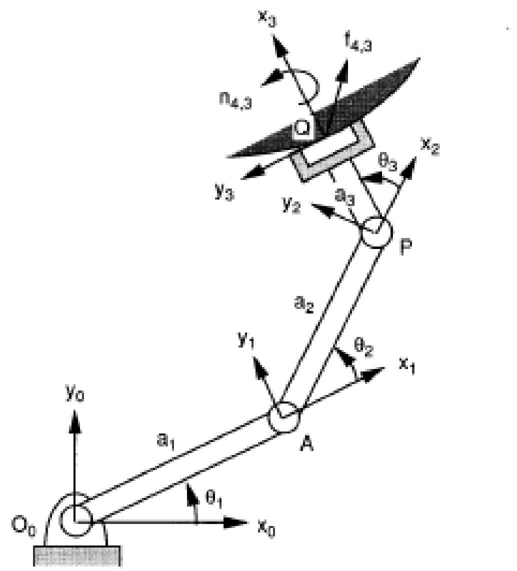


FIGURE 6.3. Planar $3R$ manipulator exerting a force $f_{4,3}$ and a moment $n_{4,3}$.

Substituting Eq. (6.18) for $i = 1$ to 3 into Eqs. (6.7) and (6.8) gives

$$\mathbf{r}_1 = {}^0R_1\,{}^1\mathbf{r}_1 = a_1 \begin{bmatrix} c\theta_1 \\ s\theta_1 \\ 0 \end{bmatrix}, \quad \mathbf{r}_{c1} = {}^0R_1\,{}^1\mathbf{r}_{c1} = -\frac{a_1}{2} \begin{bmatrix} c\theta_1 \\ s\theta_1 \\ 0 \end{bmatrix}.$$

$$\mathbf{r}_2 = {}^0R_2\,{}^2\mathbf{r}_2 = a_2 \begin{bmatrix} c\theta_{12} \\ s\theta_{12} \\ 0 \end{bmatrix}, \quad \mathbf{r}_{c2} = {}^0R_2\,{}^2\mathbf{r}_{c2} = -\frac{a_2}{2} \begin{bmatrix} c\theta_{12} \\ s\theta_{12} \\ 0 \end{bmatrix}.$$

$$\mathbf{r}_3 = {}^0R_3\,{}^3\mathbf{r}_3 = a_3 \begin{bmatrix} c\theta_{123} \\ s\theta_{123} \\ 0 \end{bmatrix}, \quad \mathbf{r}_{c3} = {}^0R_3\,{}^3\mathbf{r}_{c3} = -\frac{a_3}{2} \begin{bmatrix} c\theta_{123} \\ s\theta_{123} \\ 0 \end{bmatrix}.$$

We now apply Eqs. (6.4) and (6.5) to compute the reaction forces exerted on link 3, then proceed to link 2 and 1 in sequence. For $i = 3$, substituting $\mathbf{r}_3$, $\mathbf{r}_{c3}$, $\mathbf{f}_{4,3}$, and $\mathbf{n}_{4,3}$ into Eqs. (6.4) and (6.5) yields

$$\mathbf{f}_{3,2} = \mathbf{f}_{4,3} - m_3\mathbf{g} = \begin{bmatrix} f_x \\ f_y + m_3 g_c \\ 0 \end{bmatrix},$$

$$\mathbf{n}_{3,2} = \mathbf{n}_{4,3} + \mathbf{r}_3 \times \mathbf{f}_{3,2} - \mathbf{r}_{c3} \times m_3\mathbf{g} = \begin{bmatrix} 0 \\ 0 \\ n_{3,2z} \end{bmatrix},$$

where

$$n_{3,2z} = n_z + f_y a_3 c\theta_{123} - f_x a_3 s\theta_{123} + 0.5 m_3 g_c a_3 c\theta_{123}.$$

For $i = 2$, we substitute $\mathbf{f}_{3,2}$ and $\mathbf{n}_{3,2}$ obtained in the preceding step along with $\mathbf{r}_2$ and $\mathbf{r}_{c2}$ into Eqs. (6.4) and (6.5). As a result, we obtain

$$\mathbf{f}_{2,1} = \mathbf{f}_{3,2} - m_2\mathbf{g} = \begin{bmatrix} f_x \\ f_y + (m_2 + m_3) g_c \\ 0 \end{bmatrix},$$

$$\mathbf{n}_{2,1} = \mathbf{n}_{3,2} + \mathbf{r}_2 \times \mathbf{f}_{2,1} - \mathbf{r}_{c2} \times m_2\mathbf{g} = \begin{bmatrix} 0 \\ 0 \\ n_{2,1z} \end{bmatrix},$$

where

$$n_{2,1z} = n_z + f_y(a_2 c\theta_{12} + a_3 c\theta_{123}) - f_x(a_2 s\theta_{12} + a_3 s\theta_{123}) + 0.5 m_2 g_c a_2 c\theta_{12} + m_3 g_c(a_2 c\theta_{12} + 0.5 a_3 c\theta_{123}).$$

For $i = 1$, we substitute $\mathbf{f}_{2,1}$ and $\mathbf{n}_{2,1}$ obtained in the preceding step along with $\mathbf{r}_1$ and $\mathbf{r}_{c1}$ into Eqs. (6.4) and (6.5). This produces

$$\mathbf{f}_{1,0} = \mathbf{f}_{2,1} - m_1\mathbf{g} = \begin{bmatrix} f_x \\ f_y + (m_1 + m_2 + m_3)g_c \\ 0 \end{bmatrix},$$

$$\mathbf{n}_{1,0} = \mathbf{n}_{2,1} + \mathbf{r}_1 \times \mathbf{f}_{1,0} - \mathbf{r}_{c1} \times m_1\mathbf{g} = \begin{bmatrix} 0 \\ 0 \\ n_{1,0z} \end{bmatrix},$$

where

$$\begin{aligned} n_{1,0z} = n_z &+ f_y(a_1c\theta_1 + a_2c\theta_{12} + a_3c\theta_{123}) - f_x(a_1s\theta_1 + a_2s\theta_{12} + a_3s\theta_{123}) \\ &+ 0.5m_1g_ca_1c\theta_1 + m_2g_c(a_1c\theta_1 + 0.5a_2c\theta_{12}) \\ &+ m_3g_c(a_1c\theta_1 + a_2c\theta_{12} + 0.5a_3c\theta_{123}). \end{aligned}$$

Finally, we apply Eq. (6.17) to compute the joint torques as follows:

$$\begin{aligned} \tau_1 &= \mathbf{z}_0^T\mathbf{n}_{1,0} = n_{1,0z}, \\ \tau_2 &= \mathbf{z}_1^T\mathbf{n}_{2,1} = n_{2,1z}, \\ \tau_3 &= \mathbf{z}_2^T\mathbf{n}_{3,2} = n_{3,2z}. \end{aligned}$$

We note that in the absence of gravity, the torques and end-effector output forces are related by the following equation:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = J^T \begin{bmatrix} f_x \\ f_y \\ n_z \end{bmatrix}, \tag{6.19}$$

where

$$J = \begin{bmatrix} -(a_1s\theta_1 + a_2s\theta_{12} + a_3s\theta_{123}) & -(a_2s\theta_{12} + a_3s\theta_{123}) & -a_3s\theta_{123} \\ (a_1c\theta_1 + a_2c\theta_{12} + a_3c\theta_{123}) & (a_2c\theta_{12} + a_3c\theta_{123}) & a_3c\theta_{123} \\ 1 & 1 & 1 \end{bmatrix}.$$

Hence, in the absence of gravity, the transformation between the end-effector output forces and the joint torques is governed by the transpose of the conventional Jacobian matrix.