Benha University
College of Engineering at Banha
Mechanical Eng. Dept. **4^{th} Year Mechanics**
Subject :Automatic Control (M 1352) **May 14/2018**
Questions For Final Examination

نموذج أسئلة و اجابة

المادة : التحكم الآلى م 1352

أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

Examiner : Dr. Mohamed Elsharnoby **Time :180 min.**

Attempt all questions, Number of q 1uestions = 4, Number of pages = 2

Figure 1 shows a system with two inputs and two outputs. Derive $C_1(s)/R_1(s)$, $C_1(s)/R_2(s)$, $C_2(s)/R_1(s)$, and $C_2(s)/R_2(s)$. (In deriving outputs for $R_1(s)$, assume that $R_2(s)$ is zero, and vice versa.

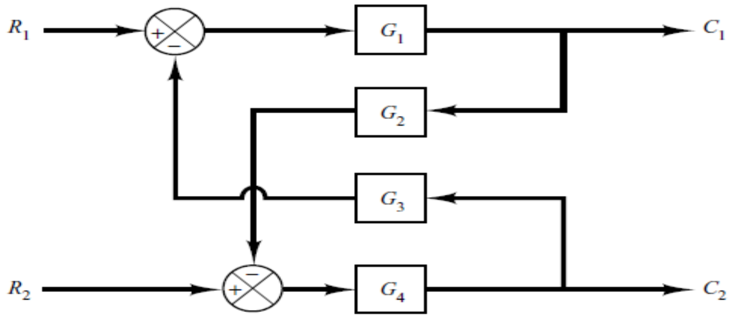

Figure 1

1-b) If the transfer function of a system is given by:

$$\frac{Y(s)}{U(s)} = \frac{2s^3 + s^2 + s + 2}{s^3 + 4s^2 + 5s + 2}$$

i) Obtain a state-space equation and output equation for the system defined by
ii) Draw a signal flow graph represents this system.

2-a) Figure 2-a shows a mechanical vibratory system. When 2 lb of force (step input) is applied to the system, the mass oscillates, as shown in Figure 2-b. Determine m, b, and k of the system from this response curve.The displacement x is measured from the equilibrium position.

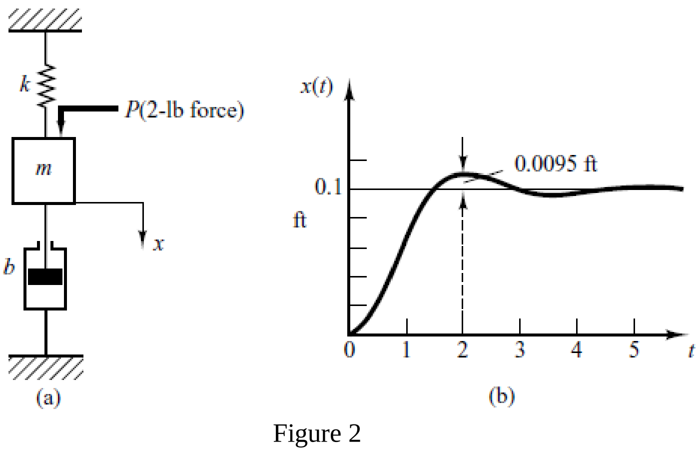

Figure 2

2-b) . Consider a feedback system shown in figure 3

For $G(s) = \frac{(4-s)}{(s-1)(s+4)}$, we use a proportional controller with K>0

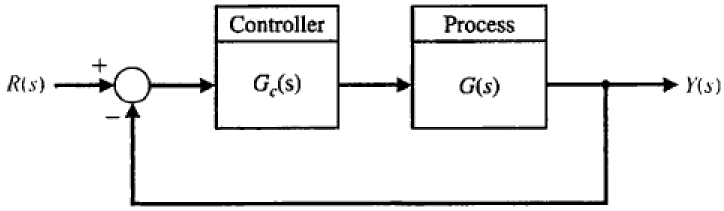

Figure 3

i) Determine the range of K for which the feedback systmis stable.
ii) Draw the Nyqusit plot for K=1.
iii) Design K>0 such that the phase margin is maximized.

Hint: You may use the following identity $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$

3-a) Sketch the root loci for the system shown in Figure 4

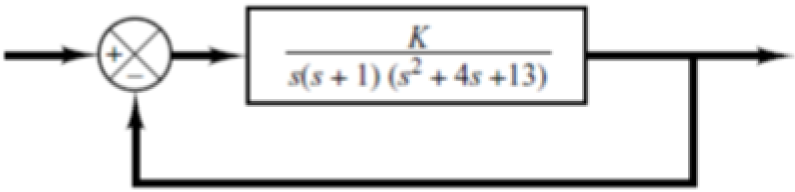

Figure 4

3-b) Consider a feedback system in figure 3, a phase-lead compensator with $G_c(s) = \frac{1+0.4s}{1+0.04s}$ is placed in series with the plant $G(s) = \frac{500}{(s+1)(s+5)(s+10)}$, compute the gain and phase margin

a. *G.M.* = ∞ dB, *P.M.* = 60°
b. *G.M.* = 20.5 dB,P.M. - 47.8°
c *G.M.* = 8.6 dB, *P.M.* = 33.6°
d. Closed-loop system is unstable
(verify your answer).

4-a) Consider the feedback system depicted in Figure 3, where

$$G(s) = \frac{1}{s(s+4)^2}$$

A suitable compensation *Gc(s)* for this system that satisfies the specifications:
(i) *P.O.* < 20%, and (ii) velocity error constant $K_v \succ 10$, is which of the following:

a. $G_c(s) = \frac{s+4}{(s+1)}$

b. $G_c(s) = \frac{160(10s+1)}{200s+1}$

c. $G_c(s) = \frac{24(s+1)}{s+4}$

d. None of the above (show your work)

4-b) Given the straight line Bode diagram of magnitude in figure 5, find the corresponding transfer function.

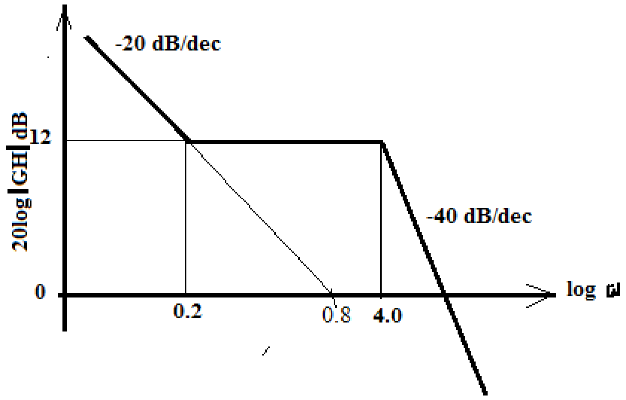

Figure 5-a

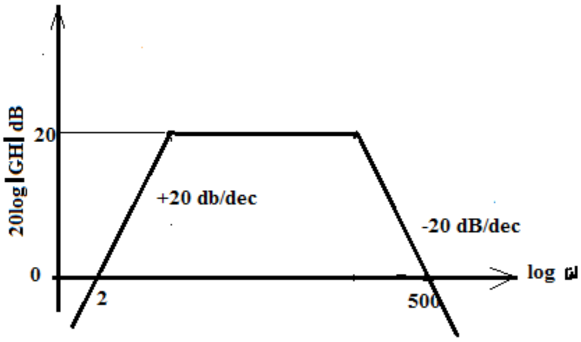

Figure 5-b

GOOD LUCK

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Model Answer of the Final Examination

Elaborated by: Dr. Mohamed Elsharnoby

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Figure 1 shows a system with two inputs and two outputs. Derive $C_1(s)/R_1(s)$, $C_1(s)/R_2(s)$,$C_2(s)/R_1(s)$, and $C_2(s)/R_2(s)$. (In deriving outputs for $R_1(s)$, assume that $R_2(s)$ is zero, and vice versa.

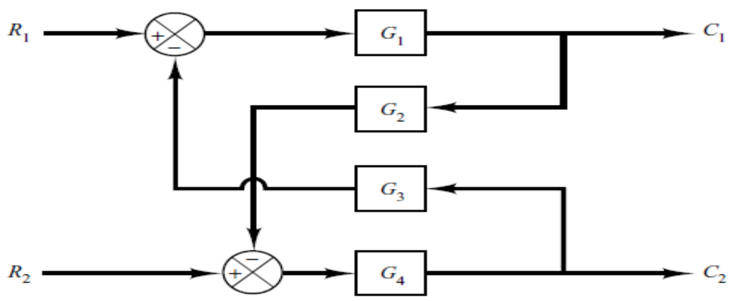


Figure 1

Solution. From the figure, we obtain

$$C_1 = G_1(R_1 - G_3C_2) \tag{2–52}$$

$$C_2 = G_4(R_2 - G_2C_1) \tag{2–53}$$

By substituting Equation (2–53) into Equation (2–52), we obtain

$$C_1 = G_1[R_1 - G_3G_4(R_2 - G_2C_1)] \tag{2–54}$$

By substituting Equation (2–52) into Equation (2–53), we get

$$C_2 = G_4[R_2 - G_2G_1(R_1 - G_3C_2)] \tag{2–55}$$

Solving Equation (2–54) for C_1, we obtain

$$C_1 = \frac{G_1R_1 - G_1G_3G_4R_2}{1 - G_1G_2G_3G_4} \tag{2–56}$$

Solving Equation (2–55) for C_2 gives

$$C_2 = \frac{-G_1G_2G_4R_1 + G_4R_2}{1 - G_1G_2G_3G_4} \tag{2–57}$$

Equations (2–56) and (2–57) can be combined in the form of the transfer matrix as follows:

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} \dfrac{G_1}{1 - G_1G_2G_3G_4} & -\dfrac{G_1G_3G_4}{1 - G_1G_2G_3G_4} \\ -\dfrac{G_1G_2G_4}{1 - G_1G_2G_3G_4} & \dfrac{G_4}{1 - G_1G_2G_3G_4} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

Then the transfer functions $C_1(s)/R_1(s)$, $C_1(s)/R_2(s)$, $C_2(s)/R_1(s)$ and $C_2(s)/R_2(s)$ can be obtained as follows:

$$\frac{C_1(s)}{R_1(s)} = \frac{G_1}{1 - G_1G_2G_3G_4}, \qquad \frac{C_1(s)}{R_2(s)} = -\frac{G_1G_3G_4}{1 - G_1G_2G_3G_4}$$

$$\frac{C_2(s)}{R_1(s)} = -\frac{G_1G_2G_4}{1 - G_1G_2G_3G_4}, \qquad \frac{C_2(s)}{R_2(s)} = \frac{G_4}{1 - G_1G_2G_3G_4}$$

Note that Equations (2–56) and (2–57) give responses C_1 and C_2, respectively, when both inputs R_1 and R_2 are present.

Notice that when $R_2(s) = 0$, the original block diagram can be simplified to those shown in Figures 2–25(a) and (b). Similarly, when $R_1(s) = 0$, the original block diagram can be simplified to those shown in Figures 2–25(c) and (d). From these simplified block diagrams we can also obtain $C_1(s)/R_1(s)$, $C_2(s)/R_1(s)$, $C_1(s)/R_2(s)$, and $C_2(s)/R_2(s)$, as shown to the right of each corresponding block diagram.

1-b)

1-b) If the transfer function of a system is given by:

$$\frac{Y(s)}{U(s)} = \frac{2s^3 + s^2 + s + 2}{s^3 + 4s^2 + 5s + 2}$$

iii) Obtain a state-space equation and output equation for the system defined by
iv) Draw a signal flow graph represents this system.

$$a_1 = 4, \quad a_2 = 5, \quad a_3 = 2$$
$$b_0 = 2, \quad b_1 = 1, \quad b_2 = 1, \quad b_3 = 2$$

Referring to Equation (2–35), we have

$$\beta_0 = b_0 = 2$$
$$\beta_1 = b_1 - a_1\beta_0 = 1 - 4 \times 2 = -7$$
$$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0 = 1 - 4 \times (-7) - 5 \times 2 = 19$$
$$\beta_3 = b_3 - a_1\beta_2 - a_2\beta_1 - a_3\beta_0$$
$$= 2 - 4 \times 19 - 5 \times (-7) - 2 \times 2 = -43$$

Referring to Equation (2–34), we define

$$x_1 = y - \beta_0 u = y - 2u$$
$$x_2 = \dot{x}_1 - \beta_1 u = \dot{x}_1 + 7u$$
$$x_3 = \dot{x}_2 - \beta_2 u = \dot{x}_2 - 19u$$

Then referring to Equation (2–36),

$$\dot{x}_1 = x_2 - 7u$$
$$\dot{x}_2 = x_3 + 19u$$
$$\dot{x}_3 = -a_3x_1 - a_2x_2 - a_1x_3 + \beta_3 u$$
$$= -2x_1 - 5x_2 - 4x_3 - 43u$$

Hence, the state-space representation of the system is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -7 \\ 19 \\ -43 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 2u$$

Another solution

$$\frac{Y(s)}{U(s)} = 2 - \frac{7s^2 + 9s + 2}{S^3 + 4s^2 + 5s + 2}$$

$$Y(s) = 2U(s) - \frac{7s^2 + 9s + 2}{S^3 + 4s^2 + 5s + 2} U(s)$$

$$Y(s) = 2U(s) + Z(s)$$

$$Z(s) = -\frac{7s^2 + 9s + 2}{S^3 + 4s^2 + 5s + 2} U(s)$$

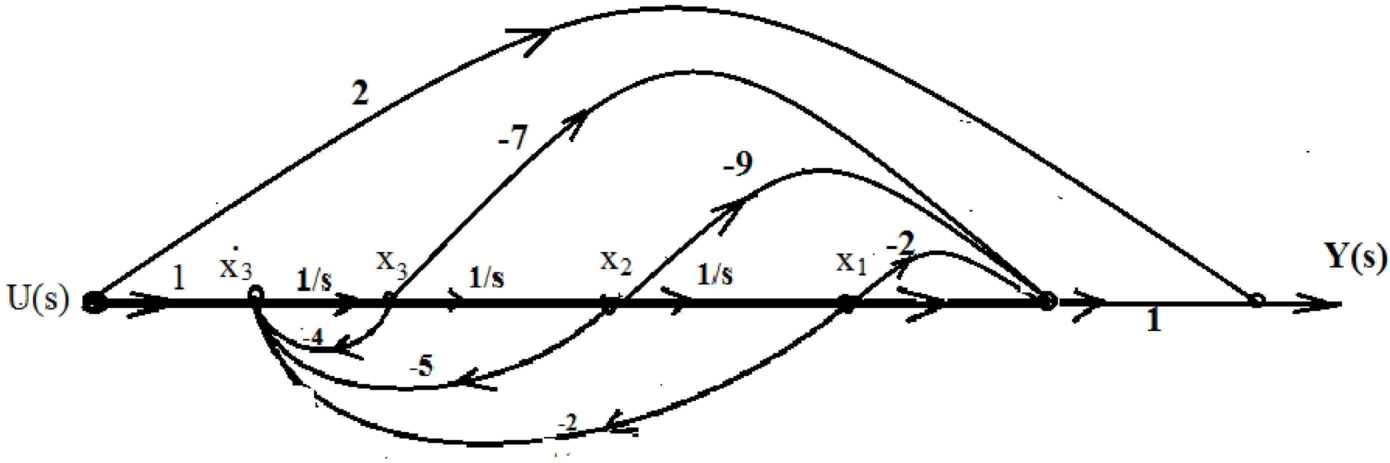
$$Z(s) = -\frac{\frac{7}{s} + \frac{9}{s^2} + \frac{2}{s^3}}{1 + \frac{4}{s} + \frac{5}{s^2} + \frac{2}{s^3}} U(s)$$

$$x_1 = z$$

$x_2 = \dot{z} = \dot{x}_1$

$x_3 = \ddot{z} = \dot{x}_2$

$\dot{x}_3 = \dddot{z} = -4x_3 - 5x_2 - 2x_1 + u$



iii) the state space representation of the system.

$$\dot{X} = AX + BU = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -2 & -5 & -4 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = CX + DU = \begin{bmatrix} -2 & -9 & -7 \end{bmatrix} X + 2U$$

2-a)

Solution. The transfer function of this system is

$$\frac{X(s)}{P(s)} = \frac{1}{ms^2 + bs + k}$$

Since

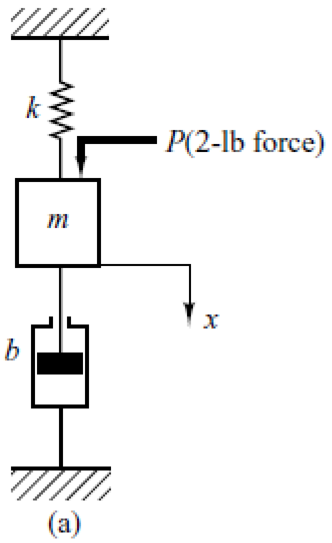
$$P(s) = \frac{2}{s}$$

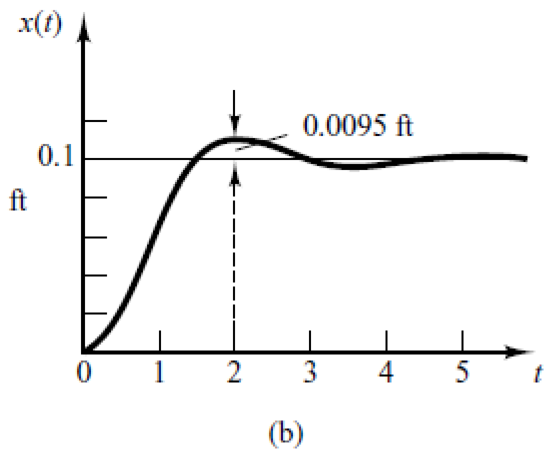
we obtain

$$X(s) = \frac{2}{s(ms^2 + bs + k)}$$

It follows that the steady-state value of x is

$$x(\infty) = \lim_{s \to 0} sX(s) = \frac{2}{k} = 0.1 \text{ ft}$$





(a) Mechanical vibratory system; (b) step-response curve.

Hence

$$k = 20 \text{ lb}_f/\text{ft}$$

Note that $M_p = 9.5\%$ corresponds to $\zeta = 0.6$. The peak time t_p is given by

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{0.8\omega_n}$$

The experimental curve shows that $t_p = 2$ sec. Therefore,

$$\omega_n = \frac{3.14}{2 \times 0.8} = 1.96 \text{ rad/sec}$$

Since $\omega_n^2 = k/m = 20/m$, we obtain

$$m = \frac{20}{\omega_n^2} = \frac{20}{1.96^2} = 5.2 \text{ slugs} = 167 \text{ lb}$$

(Note that 1 slug $= 1 \text{ lb}_f\text{-sec}^2/\text{ft}$.) Then b is determined from

$$2\zeta\omega_n = \frac{b}{m}$$

or

$$b = 2\zeta\omega_n m = 2 \times 0.6 \times 1.96 \times 5.2 = 12.2 \text{ lb}_f/\text{ft/sec}$$

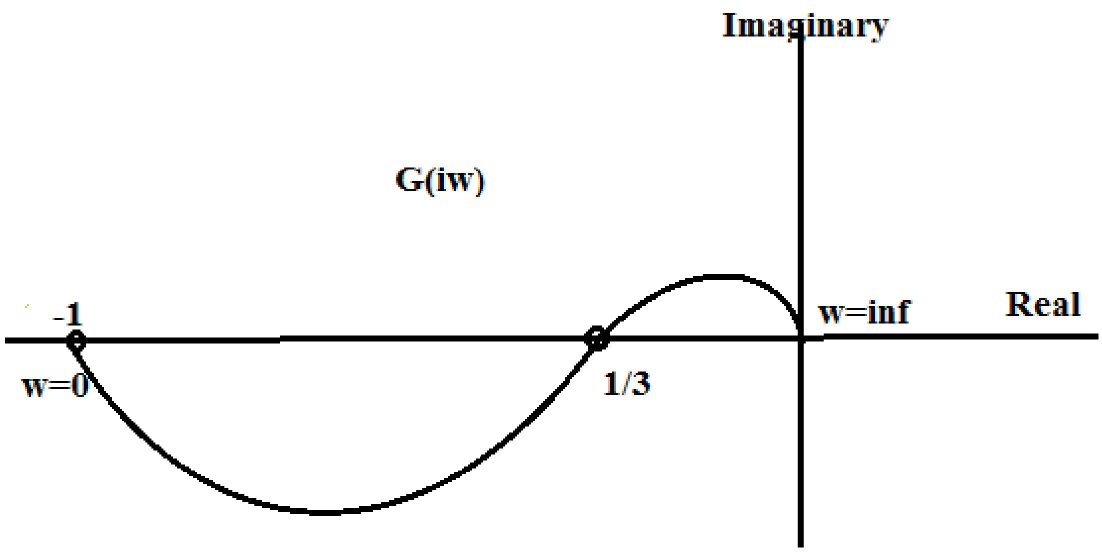
2-b)

i) The characteristic equation is given by 1+KG = 0
This is reduced to $S^2 + (3\text{-}k)S + 4(K\text{-}1)$
All coefficient should be +ve
$\therefore 1 \le K \le 3$

ii) $G_P(i\omega) = \dfrac{4 - i\omega}{(-4-\omega^2) + 3i\omega} = \dfrac{-4 + i\omega}{(4+\omega^2) - 3i\omega}$

$$G_P(i\omega) = \frac{-16 - 7\omega^2 + i(\omega^3 - 8\omega)}{(4+\omega^2)^2 + 9\omega^2}$$

Intersection with real for ω = 0 or $\sqrt{8}$, at -1 and – 0.3333 respectively



$\phi = \tan^{-1} \frac{\omega^3 - 8\omega}{-16 - 7\omega^2}$

The maximum phase will not change with K but the value of $|G(i\omega)|$

Let $\phi = \tan^{-1} x$ for max ϕ, $\frac{d\phi}{d\omega} = \frac{d\phi}{dx}\frac{dx}{d\omega} = \frac{1}{1+x^2}\frac{dx}{d\omega} = 0 \Rightarrow \frac{dx}{d\omega} = 0$

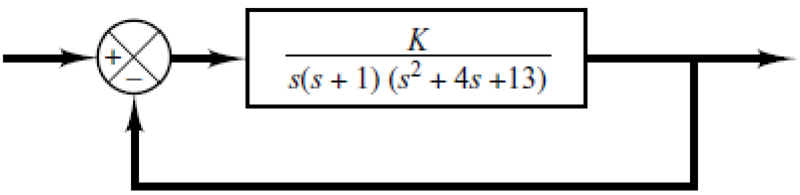
$\frac{dx}{d\omega} = 0$ for $7\omega^4 + 104\omega^2 - 128 = 0 \Rightarrow \omega^2 = \frac{8}{7}, \Rightarrow \omega = 1.069 s^{-1}$

$G(i1.069) = -0.6533 - 0.1995i$

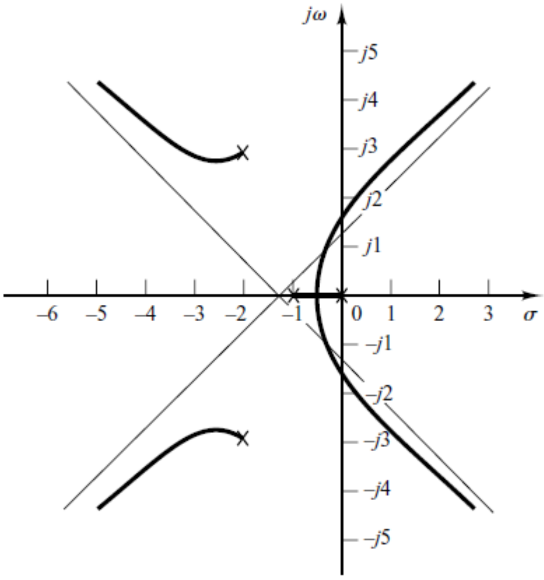
$|G(i\omega)| = 0.68308 \quad \therefore k = 1.464$ to get tha max phase margin which equal to $\phi = \tan^{-1} \frac{0.1995}{0.6533}$

3-a) The open-loop poles are located at s=0, s=–1, s=–2+j3, and s=–2-j3.A root locus exists on the real axis between points s=0 and s=–1. The angles of the asymptotes are found as follows:

$$\text{Angles of asymptotes} = \frac{\pm 180°(2k+1)}{4} = 45°, -45°, 135°, -135°$$



(a)



(b)

(a) Control system; (b) root-locus plot.

The intersection of the asymptotes and the real axis is found from

$$s = -\frac{0 + 1 + 2 + 2}{4} = -1.25$$

The breakaway and break-in points are found from $dK/ds = 0$. Noting that

$$K = -s(s + 1)(s^2 + 4s + 13) = -(s^4 + 5s^3 + 17s^2 + 13s)$$

we have

$$\frac{dK}{ds} = -(4s^3 + 15s^2 + 34s + 13) = 0$$

from which we get

$$s = -0.467, \qquad s = -1.642 + j2.067, \qquad s = -1.642 - j2.067$$

Point $s = -0.467$ is on a root locus. Therefore, it is an actual breakaway point. The gain values K corresponding to points $s = -1.642 \pm j2.067$ are complex quantities. Since the gain values are not real positive, these points are neither breakaway nor break-in points.

The angle of departure from the complex pole in the upper-half s plane is

$$\theta = 180^\circ - 123.69^\circ - 108.44^\circ - 90^\circ$$

or

$$\theta = -142.13°$$

Next we shall find the points where root loci may cross the $j\omega$ axis. Since the characteristic equation is

$$s^4 + 5s^3 + 17s^2 + 13s + K = 0$$

by substituting $s = j\omega$ into it we obtain

$$(j\omega)^4 + 5(j\omega)^3 + 17(j\omega)^2 + 13(j\omega) + K = 0$$

or

$$\left(K + \omega^4 - 17\omega^2\right) + j\omega\left(13 - 5\omega^2\right) = 0$$

from which we obtain

$$\omega = \pm\ 1.6125, \qquad K = 37.44 \qquad \text{or} \qquad \omega = 0, \qquad K = 0$$

The root-locus branches that extend to the right-half s plane cross the imaginary axis at $\omega = \pm 1.6125$. Also, the root-locus branch on the real axis touches the imaginary axis at $\omega = 0$. Figure 6–68(b) shows a sketch of the root loci for the system. Notice that each root-locus branch that extends to the right-half s plane crosses its own asymptote.

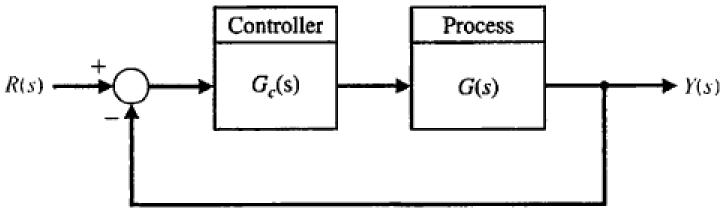
. Consider a feedback system in which a phase-lead compensator

$$G_c(s) = \frac{1 + 0.4s}{1 + 0.04s}$$

is placed in series with the plant

$$G(s) = \frac{500}{(s + 1)(s + 5)(s + 10)}.$$

The feedback system is a negative unity feedback control system shown in Figure 10.43.
Compute the gain and phase margin.



a. $G.M. = \infty$ dB, $P.M. = 60°$
b. $G.M. = 20.5$ dB,P.M. - 47.8°
c $G.M. = 8.6$ dB, $P.M. = 33.6°$
d. Closed-loop system is unstable
(verify your answer).

4-a)Consider the feedback system depicted in Figure 10.43, where

$$G(s) = \frac{1}{s(s + 4)^2}.$$

A suitable compensation $G_c(s)$ for this system that satisfies the specifications:
(i) $P.O. < 20\%$, and (ii) velocity error constant K_v a 10, is which of the

following:

a. $G_c(s) = \dfrac{s+4}{(s+1)}$

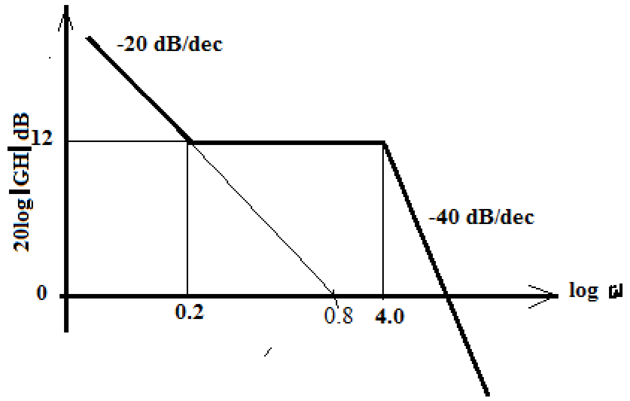
b. $G_c(s) = \dfrac{160(10s+1)}{200s+1}$

c. $G_c(s) = \dfrac{24(s+1)}{s+4}$

d. None of the above

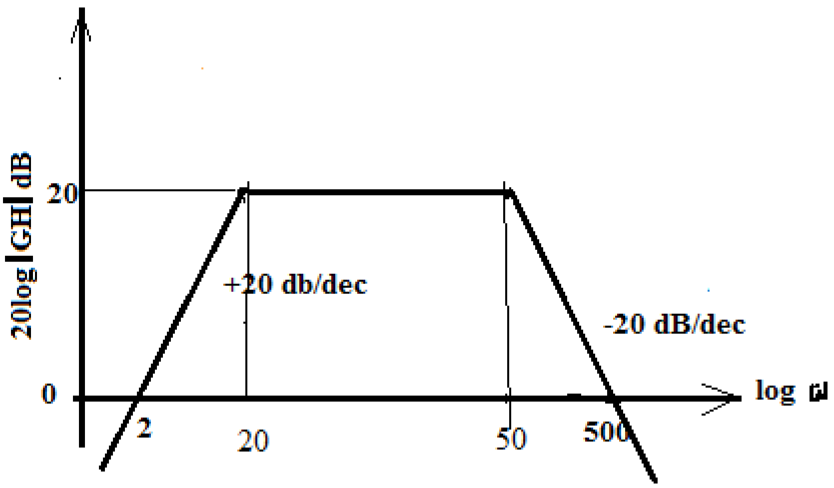
(verify your answer)

The correct answer is (b)



(a)

For (a) $T(s) = \dfrac{0.8(\frac{s}{0.2}+1)}{s(\frac{s}{4}+1)^2} = \dfrac{(4s+0.8)}{\frac{s}{16}(s+4)^2} = \dfrac{64(s+0.2)}{s(s+4)^2}$



For (b) $T(s) = \dfrac{s}{2(\frac{s}{20}+1)(\frac{s}{50}+1)} = \dfrac{500s}{(s+20)(s+50)}$