



<b>Benha University</b>	<b>Time: 3hour</b>		
<b>Benha Faculty of Engineering</b>	<b>Forth Year 21-5-2016</b>		
<b>Subject: Fields(Elect. Machines) (E 312)</b>	<b>Elect. Eng. Dept. تخلفات</b>		

**Solve & draw as much as you can (questions in two pages)**

**Question (1)**

**[20] Points**

1.a) Sketch and explain the torque-speed characteristics of 3-phase induction motor?

1.b) Explain the construction of 3-phase induction motor?

1.c) Explain the starting of 3-phase induction motor?

1.d) A three phase 220V, 60Hz, 6 poles, 10HP, wye-connected induction motor has a stator impedance of  $(0.344+j0.498) \Omega$ /phase and  $(0.147+j0.224)\Omega$ /phase of the rotor winding referred to the stator side. The exciting branch impedance viewed from the stator side is  $(j12. \Omega)$ . The no load loss=262 watt and may be assumed constant and a slip of 0.02.

i-Draw the equivalent circuit?

ii- Determine speed; mechanical power developed; developed torque; and efficiency?

**Question (2)**

**[25] Points**

2.a) Sketch and explain the torque-speed characteristics of DC motor?

2.b) Explain the construction of DC motor?

2.c) Explain the starting of DC motor?

2.d) A 120 V shunt DC motor has the following parameters  $R_a=0.4 \Omega$ ,  $R_f=120 \Omega$  and rotational losses are 240 watt on full load the line current is 19.5A and the motor runs at 1200rpm.

i-Draw the equivalent circuit?

ii- Determine: the armature current, the field current, shaft speed, mechanical power developed, developed torque, and efficiency?

**P.T.O.**

2.e) A separately excited DC generator has an open circuit terminal voltage of 144V. When loaded by resistive load the voltage across the load is 120V. The armature resistance is  $0.52 \Omega$  and the field supply voltage is 220V and field resistance is  $147 \Omega$ .

**Determine** (i) armature current and field current (ii) the efficiency?

**Question (3)**

**[15] Points**

3.a) Explain the construction of operation of 3-phase synchronous motor?

3.b) - A three phase 460V, 60Hz, 6 poles, wye-connected cylindrical rotor synchronous motor has a synchronous reactance of  $2 \Omega/\text{phase}$ .  $R_s$  is negligible and  $I_s=22\text{A}/\text{phase}$  and unity p.f.

i-Draw the equivalent circuit?

ii-Find the rotor speed and torque angle?

iii-Find the  $P_{\text{out}}$  and the maximum torque?

*With my Best Wishes*

Answer

**Question (1)**

**[15] Points**

1.a) Sketch and explain the torque-speed characteristics of a 3-phase induction motor?

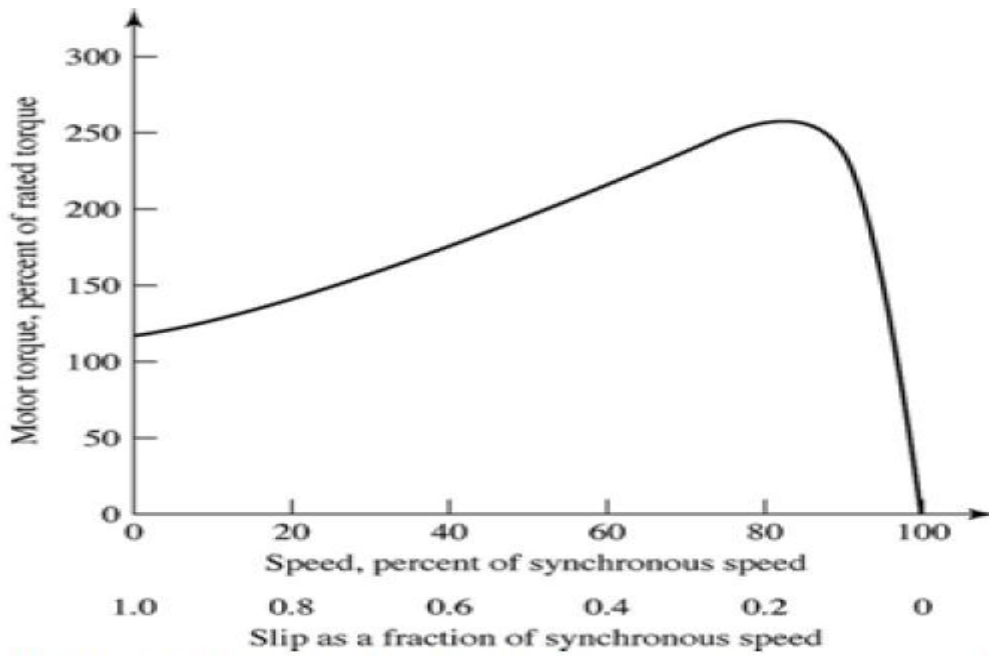


Figure 6.4 Typical induction-motor torque-speed

$$P_{\text{gap}} = n_{\text{ph}} I_2^2 \left( \frac{R_2}{s} \right)$$

$$P_{\text{rotor}} = n_{\text{ph}} I_{2s}^2 R_2$$

$$P_{\text{rotor}} = n_{\text{ph}} I_2^2 R_2$$

$$P_{\text{mech}} = P_{\text{gap}} - P_{\text{rotor}} = n_{\text{ph}} I_2^2 \left( \frac{R_2}{s} \right) - n_{\text{ph}} I_2^2 R_2$$

$$P_{\text{mech}} = n_{\text{ph}} I_2^2 R_2 \left( \frac{1-s}{s} \right)$$

$$P_{\text{mech}} = (1-s)P_{\text{gap}}$$

$$P_{\text{rotor}} = sP_{\text{gap}}$$

$$s = \frac{n_s - n}{n_s}$$

The slip is often expressed in percent.

$n$  : rotor speed in rpm

$$n = (1-s)n_s$$

$\omega_m$  : mechanical angular velocity

$$\omega_m = (1-s)\omega_s$$

**1.b)** An induction machine is one in which alternating current is supplied to the stator directly and to the rotor by induction or transformer action from the stator.

**Construction:** 1-stator 2-rotor

1-The stator winding is excited from a balanced three-phase source and produces a magnetic field in the air gap rotating at synchronous speed.

2-The rotor winding may one of two types.

a-A wound rotor is built with a three-phase winding similar to, and wound with the same number of poles as, the stator. The rotor terminals are available external to the motor.

B-A squirrel-cage rotor has a winding consisting of conductor bars embedded in slots in the rotor iron and short-circuited at each end by conducting end rings. It is the most commonly used type of motor in sizes ranging from fractional horsepower on up.

The difference between synchronous speed and the rotor speed is commonly referred to as the slip of the rotor.

The fractional slip  $s$  is 
$$s = \frac{n_s - n}{n_s} \quad (6.1)$$

$n$  : rotor speed in rpm 
$$n = (1 - s)n_s \quad (6.2)$$

$\omega_m$  : mechanical angular velocity 
$$\omega_m = (1 - s)\omega_s \quad (6.3)$$

$f_r$  : the frequency of induced voltages, the slip frequency  $f_r = s f_e$

A wound-rotor induction machine can be used as a frequency changer.

1-The rotor currents produce an air-gap flux wave that rotates at synchronous speed and in synchronism with that produced by the stator currents.

2-With the rotor revolving in the same direction of rotation as the stator field, the rotor currents produce a rotating flux wave rotating at  $sn_s$  with respect to the rotor in the forward direction.

3-With respect to the stator, the speed of the flux wave produced by the rotor currents (with frequency  $sf_e$ ) equals  $= sn_s + n = sn_s + n_s(1 - s) = n_s$

4-Because the stator and rotor fields each rotate synchronously, they are stationary with respect to each other and produce a steady torque, thus maintaining rotation of the rotor. Such torque is called an asynchronous torque.

$$T = -\frac{\pi}{2} \left( \frac{\text{poles}}{2} \right)^2 \Phi_{sr} F_r \sin \delta_r = - KI_r \sin \delta_r \quad (6.6)$$

$I_r$  : the rotor current

$\delta_r$  : the angle by which the rotor mmf wave leads the resultant air-gap mmf

An induction machine is one in which alternating current is supplied to the stator directly and to the rotor by induction or transformer action from the stator.

- The stator winding is excited from a balanced polyphase source and produces a magnetic field in the air gap rotating at synchronous speed.
- The rotor winding may one of two types.
  - ➔ A wound rotor is built with a polyphase winding similar to, and wound with the same number of poles as, the stator. The rotor terminals are available external to the motor.
  - ➔ A squirrel-cage rotor has a winding consisting of conductor bars embedded in slots in the rotor iron and short-circuited at each end by conducting end rings. It is the most commonly used type of motor in sizes ranging from fractional horsepower on up.
- The difference between synchronous speed and the rotor speed is commonly referred to as the slip of the rotor. The fractional slip  $s$  is

$$s = \frac{n_s - n}{n_s} \quad (6.1)$$

1.c) starting with 1-auto-transformer 2-series resistors 3-series impedance

4-delta-star connection 5-electronic starter using inverter or AC converter

### 35.9. Starting of Induction Motors

It has been shown earlier that a plain induction motor is similar in action to a polyphase transformer with a short-circuited rotating secondary. Therefore, if normal supply voltage is applied to the stationary motor, then, as in the case of a transformer, a very large initial current is taken by the primary, at least, for a short while. It would be remembered that exactly similar conditions exist in the case of a d.c. motor, if it is thrown directly across the supply lines, because at the time of starting it, there is no back e.m.f. to oppose the initial inrush of current.

Induction motors, when direct-switched, take five to seven times their full-load current and develop only 1.5 to 2.5 times their full-load torque. This initial excessive current is objectionable because it will produce large line-voltage drop that, in turn, will affect the operation of other electrical equipment connected to the same lines. Hence, it is not advisable to line-start motors of rating above 25 kW to 40 kW.

It was seen in Art. 34.15 that the starting torque of an induction motor can be improved by increasing the resistance of the rotor circuit. This is easily feasible in the case of slip-ring motors but not in the case of squirrel-cage motors. However, in their case, the initial in-rush of current is controlled by applying a reduced voltage to the stator during the starting period, full normal voltage being applied when the motor has run up to speed.

### 35.10. Direct-switching or Line starting of Induction Motors

It has been shown earlier that

$$\text{Rotor input} = 2 \pi N_s T = k T \quad \text{---Art. 34.36}$$

Also, rotor Cu loss =  $s \times$  rotor input

$$\therefore 3 I_2^2 R_2 = s \times k T \quad \therefore T \propto I_2^2 / s \quad (\text{if } R_2 \text{ is the same})$$

$$\text{Now } I_2 \propto I_1 \quad \therefore T \propto I_1^2 / s \quad \text{or } T = K I_1^2 / s$$

$$\text{At starting moment } s = 1 \quad \therefore T_{st} = K I_{st}^2 \quad \text{where } I_{st} = \text{starting current}$$

$$\text{If } I_f = \text{normal full-load current and } s_f = \text{full-load slip}$$

$$\text{then } T_f = K I_f^2 / s_f \quad \therefore \frac{T_{st}}{T_f} = \left( \frac{I_{st}}{I_f} \right)^2 \cdot s_f$$

When motor is direct-switched onto normal voltage, then starting current is the short-circuit current  $I_{sc}$ .

$$\therefore \frac{T_{st}}{T_f} = \left( \frac{I_{sc}}{I_f} \right)^2 \cdot s_f = a^2 \cdot s_f \quad \text{where } a = I_{sc} / I_f$$

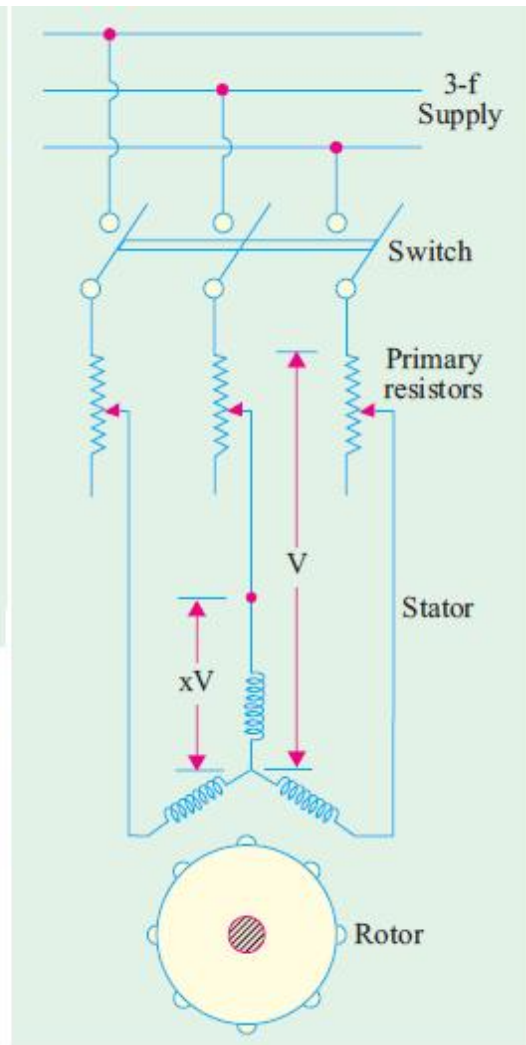
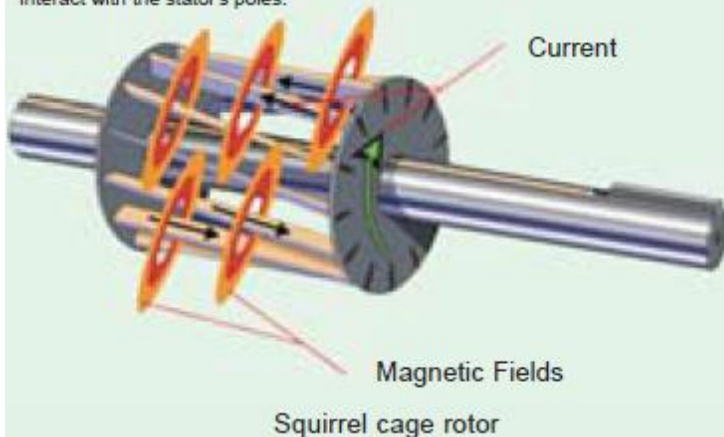
### 35.11. Squirrel-cage Motors

#### (a) Primary resistors

Their purpose is to drop some voltage and hence reduce the voltage applied across the motor terminals. In this way, the initial current drawn by the motor is reduced. However, it should be noted that whereas current varies directly as the voltage, the torque varies as square of applied voltage\*

### Squirrel Cage Rotor

When the stator's moving magnetic field cuts across the rotor's conductor bars, it induces voltage in them. This voltage produces current, which circulates through the bars and around the rotor end ring. This current in turn produces magnetic fields around each rotor bar. The continuously changing stator magnetic field results in a continuously changing rotor field. The rotor becomes an electromagnet with continuously alternating poles, which interact with the stator's poles.



(Art 34.17). If the voltage applied across the motor terminals is reduced by 50%, starting current is reduced by 50%, but torque is reduced to 25% of the full-voltage value.

By using primary resistors (Fig. 35.18), the applied voltage/phase can be reduced by a fraction 'x' (and it additionally improves the power factor of the line slightly).

$$I_{st} = x I_{sc} \quad \text{and} \quad T_{st} = x^2 T_{sc}$$

As seen from Art 35.10, above,

$$\frac{T_{st}}{T_{sc}} = \left( \frac{I_{st}}{I_{sc}} \right)^2 = \left( \frac{x I_{sc}}{I_{sc}} \right)^2 = x^2$$

### (b) Auto-transformers

Such starters, known variously as *auto-starters* or *compensators*, consist of an auto-transformer, with necessary switches. We may use either two auto-transformers connected as usual [Fig. 35.19 (b)] or 3 auto-transformers connected in open delta [Fig. 35.19



(a)]. *This method can be used both for star-and delta-connected motors.* As shown in Fig. 35.20 with starting connections, a reduced voltage is applied across the motor terminals. When the motor has ran up to say, 80% of its normal speed, connections are so changed that auto-transformers are cut out and full supply voltage is applied across the motor. The switch making these changes from 'start' to 'run' may be airbreak (for small motors) or may be oil-immersed (for large motors) to reduce sparking. There is also provision for no-voltage and over-load protection, along with a time-delay device, so that momentary interruption of voltage or momentary over-load do not disconnect the motor from supply line. Most of the auto-starters are provided with 3 sets of taps, so as to reduce voltage to 80, 65 or 50 per cent of the line voltage, to suit the local conditions of supply. The

**1.d)** A three phase 220V, 60Hz, 6 poles, 10HP, wye-connected induction motor has a stator impedance of  $(0.344 + j0.498) \Omega/\text{phase}$  and  $(0.147 + j0.224) \Omega/\text{phase}$  of the rotor winding referred to the stator side. The exciting branch impedance viewed from the stator side is  $(j12.6 \Omega)$ . The no load loss = 262 watt and may be assumed constant and a slip of 0.02.

i-Draw the equivalent circuit?

ii- Determine shaft speed; mechanical power developed; developed torque; and efficiency?

$$V_{ph} = 220/\sqrt{3} = 127V \angle 0V, Z_m = X_m = 12.6 \angle 90\Omega, I_M = 127/j 12.6 = -j10.1 = 10.1 \angle -90A$$

$$Z_{eq} = R_{eq} + jX_{eq} = 0.344 + j0.498 + 0.147/0.02 + j0.224 = 7.7 + j0.722 = 7.7 \angle 5.35\Omega,$$

$$I_r = V_1/Z_{eq} = 127 \angle 0/7.7 \angle 5.35 = 16.5 \angle -5.35A$$

$$I_s = I_M + I_r = 22.33 - j2.88 - j10.1 = 22.33 - j12.98 = 25.81 \angle -30.13 A, pf = \cos 30.13 = 0.865$$

$$n_s = 120 * 60/6 = 1200rpm, \omega_s = 1200 * \pi/30 = 40\pi = 125.7rad/s$$

$$n_r = (1-S)n_s = (1-0.02)1200 = 1176.4rpm, \omega_r = 1176 * \pi/30 = 123.145rad/s$$

$$\text{Core losses} + \text{rotational losses} = 262W$$

$$\text{Copper losses} = 3 I_L^2 (R_1 + R_2') = 3 * 16.5^2 * (0.344 + 0.147) = 408W$$

$$P_{mech} = 3 * 16.5^2 * 0.147(1-0.02)/0.02 = 5883, T_{mech} = 5883/123.145 = 47.8Nm$$

$$\text{Output power} = P_{mech} - \text{rotational losses} = 5883 - 262 = 5621W$$

$$T_{out} = 5621/123.145 = 45.7Nm$$

$$\eta = P_{out}/P_{in} = P_{out}/(P_{out} + \text{losses}) = 5621/(5621 + 262 + 408) = 0.9834$$

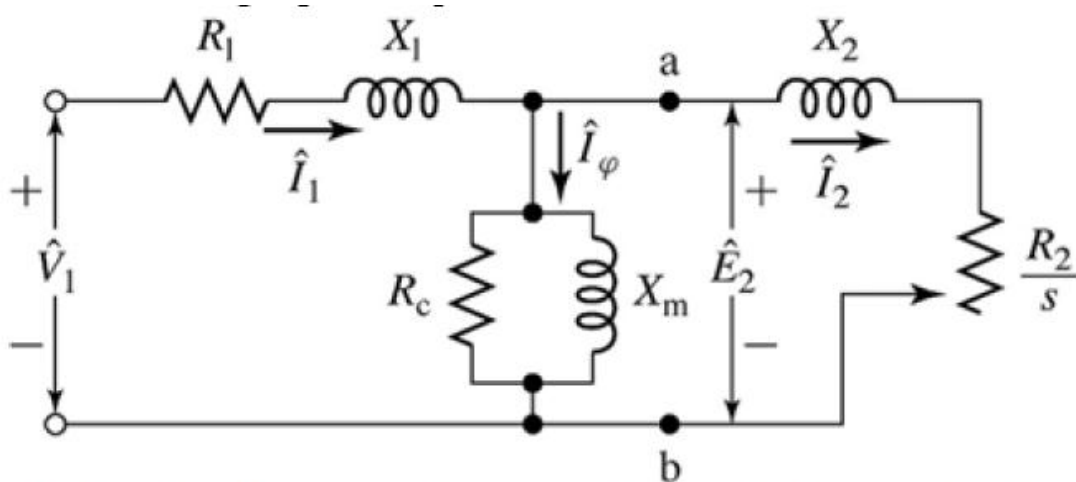


Figure 6.9 Single-phase equivalent circuit for a polyphase induction motor.

**Question (2)**

**[25] Points**

2.a) Sketch and explain the torque-speed characteristics of DC motor?

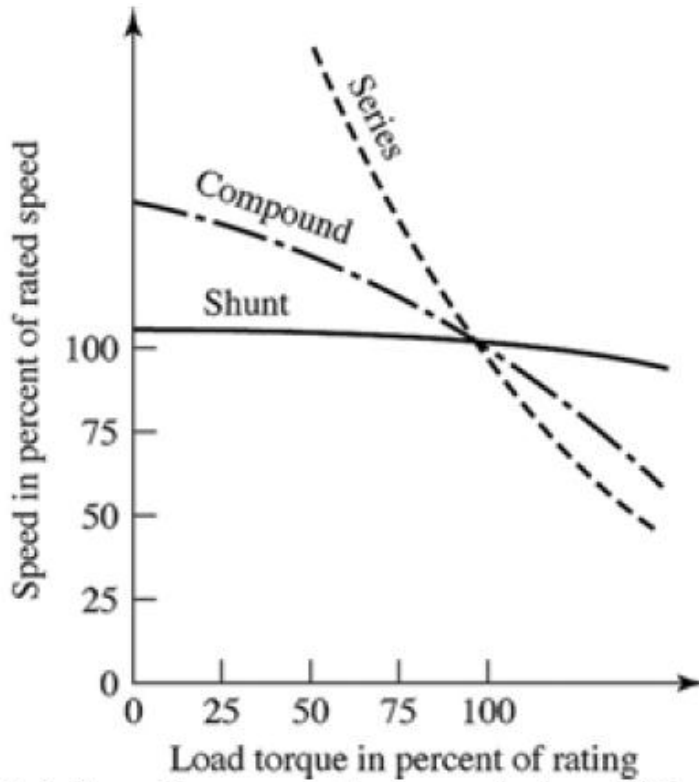


Figure 7.6 Speed-torque characteristics of dc motors.

2.b) Explain the construction of DC motor?

- ⌚ Dc machines are characterized by their versatility.
- ⌚ By means of various combinations of shunt-, series-, and separately-excited field windings they can be designed to display a wide variety of volt-ampere or speed-torque characteristics for both dynamic and steady-state operation.

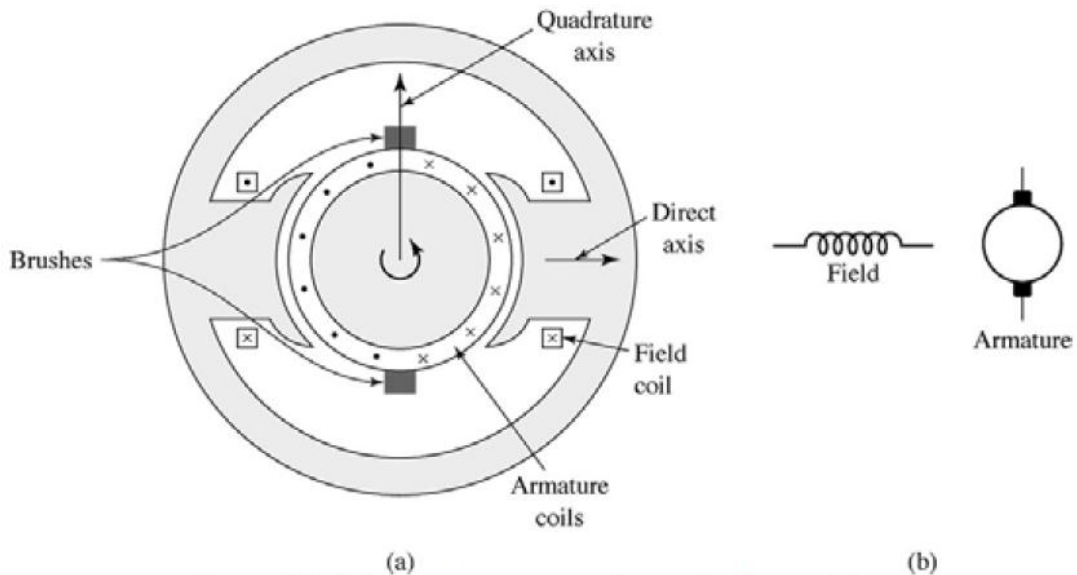


Figure 7.1 Schematic representations of a dc machine.

$$T_{\text{mech}} = K_a \Phi_d i_a$$



$K_a$  : a constant determined by the design of the winding, the winding constant

$i_a$  = current in external armature circuit

$C_a$  = total number of conductors in armature winding,

$m$  = number of parallel paths through winding

The rectified voltage  $e_a$  between brushes, known also as the speed voltage, is

$$e_a = K_a \Phi_d \omega_m$$

Note that the electric power equals the mechanical power.

$$e_a i_a = T_{\text{mech}} \omega_m$$

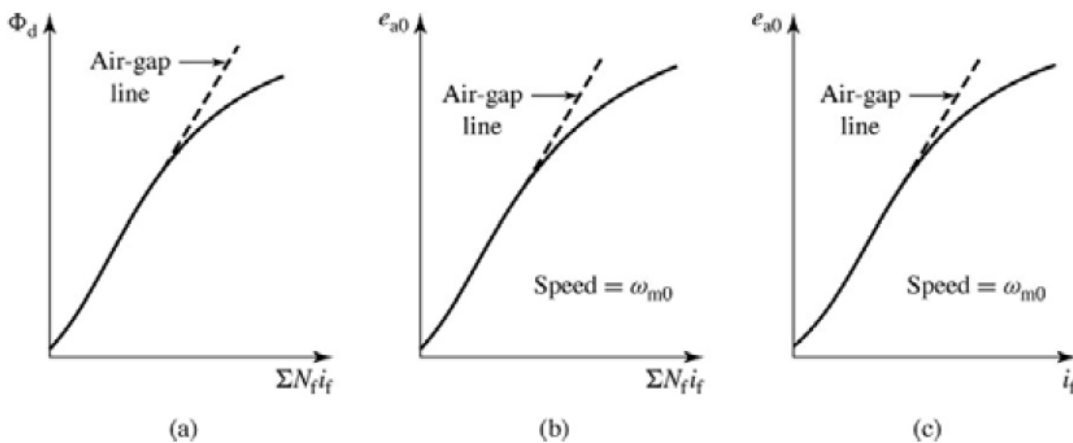


Figure 7.3 Typical form of magnetization curves of a dc machine.

Various methods of excitation of the field windings are shown in Fig. 7.4.

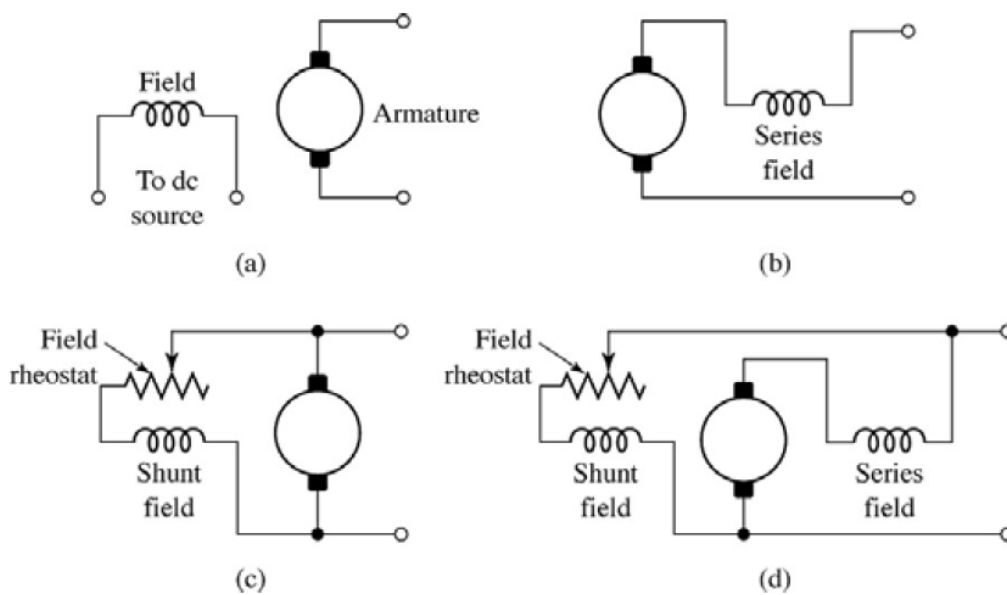


Figure 7.4 Field-circuit connections of dc machines:  
 (a) separate excitation, (b) series, (c) shunt, (d) compound.

$$V_a = E_a - I_a R_a$$

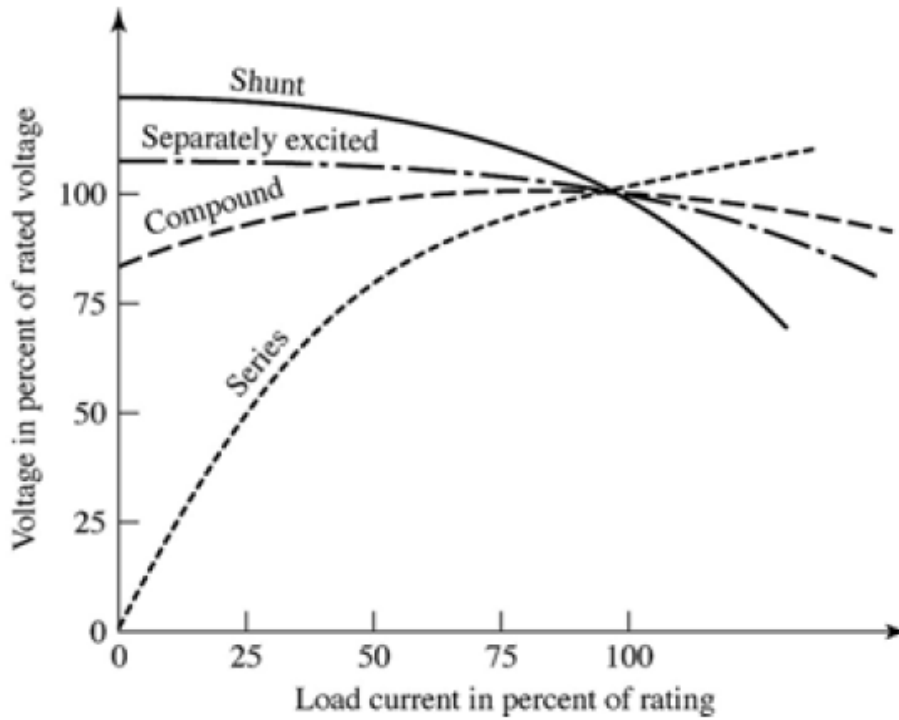


Figure 7.5 Volt-ampere characteristics of dc generators.

Any of the methods of excitation used for generators can also be used for motors.

→ Typical steady-state dc-motor speed-torque characteristics are shown in Fig. 7.6, in which it is assumed that the motor terminals are supplied from a constant-voltage source.

→ In a motor the relation between the emf  $E_a$  generated in the armature and the armature terminal voltage  $V_a$  is

$$V_a = E_a + I_a R_a \quad (7.11)$$

$$I_a = \frac{V_a - E_a}{R_a} \quad (7.12)$$

→ The application advantages of dc machines lie in the variety of performance characteristics offered by the possibilities of shunt, series, and compound excitation.

## 7.4 Analytical Fundamentals: Electric-Circuit Aspects

Analysis of dc machines: electric-circuit and magnetic-circuit aspects

➤ Torque and power:

The electromagnetic torque  $T_{\text{mech}}$

$$T_{\text{mech}} = K_a \Phi_d I_a$$

The generated voltage  $E_a$

$$E_a = K_a \Phi_d \omega_m$$

$$K_a = \frac{\text{poles} C_a}{2\pi m}$$

$E_a I_a$  : electromagnetic power

$$T_{\text{mech}} = \frac{E_a I_a}{\omega_m} = K_a \Phi_d I_a$$

Voltage and current (Refer to Fig. 7.12.):

$V_a$  : the terminal voltage of the armature winding

$V_t$  : the terminal voltage of the dc machine, including the voltage drop across the series-connected field winding

$V_a = V_t$  if there is no series field winding

$R_a$  : the resistance of armature,  $R_s$  : the resistance of the series field

$$V_a = E_a \pm I_a R_a$$

$$V_t = E_a \pm I_a (R_a + R_s)$$

$$I_L = I_a \pm I_f$$

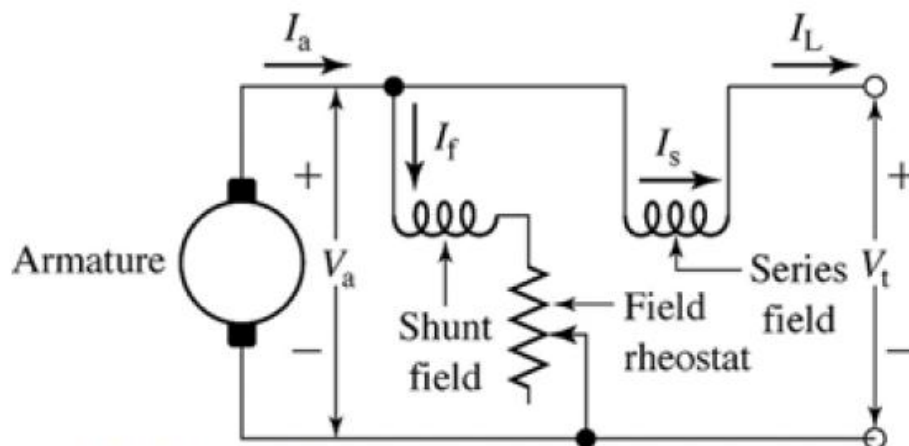


Figure 7.13 Short-shunt compound-generator connections.

**1.c)** starting with 1-series resistors 2-voltage divider 3-electronic starter using rectifier or DC chopper

### 30.20. Shunt Motor Starter

The face-plate box type starters used for starting shunt and compound motors of ordinary industrial capacity are of two kinds known as three-point and four-point starters respectively.

### 30.21. Three-point Starter

The internal wiring for such a starter is shown in Fig. 30.39 and it is seen that basically the connections are the same as in Fig. 30.37 except for the additional protective devices used here. The three terminals of the starting box are marked *A*, *B* and *C*. One line is directly connected to one armature terminal and one field terminal which are tied together. The other line is connected to point *A* which is further connected to the starting arm *L*, through the overcurrent (or overload) release *M*.

To start the motor, the main switch is first closed and then the starting arm is slowly moved to the right. As soon as the arm makes contact with stud No. 1, the field circuit is directly connected across the line and at the same time full starting resistance *R*, is placed in series with the armature. The starting current drawn by the armature =  $V/(R_a + R_s)$  where  $R_s$  is the starting

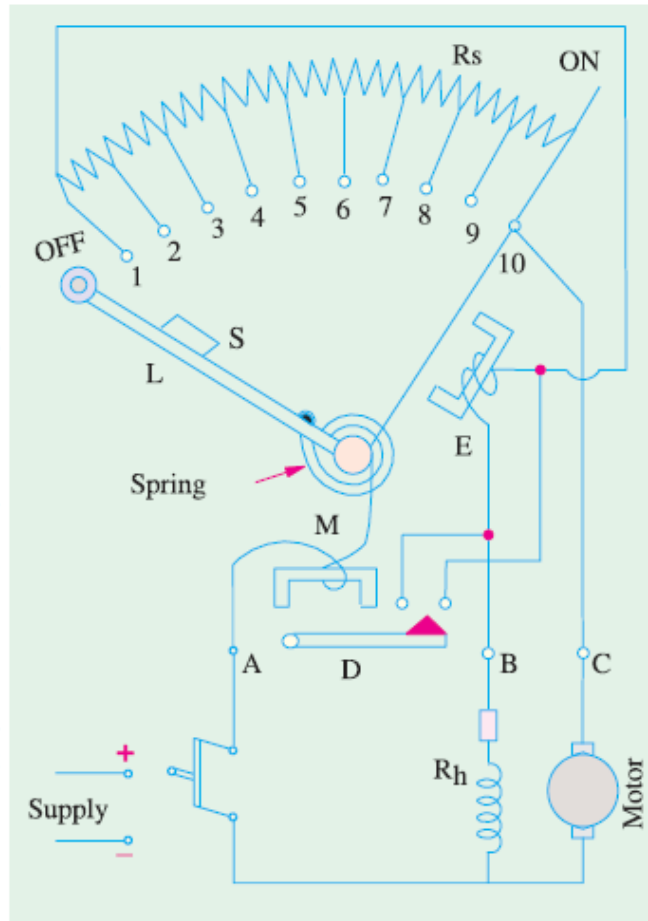


Fig. 30.39

### 30.23. Starter and Speed-control Rheostats

Sometimes, for convenience, the field rheostat is also contained within the starting box as shown in Fig. 30.43. In this case, two arms are used. There are two rows of studs, the lower ones being connected to the armature. The inside starting arm moves over the lower studs on the starting resistor,



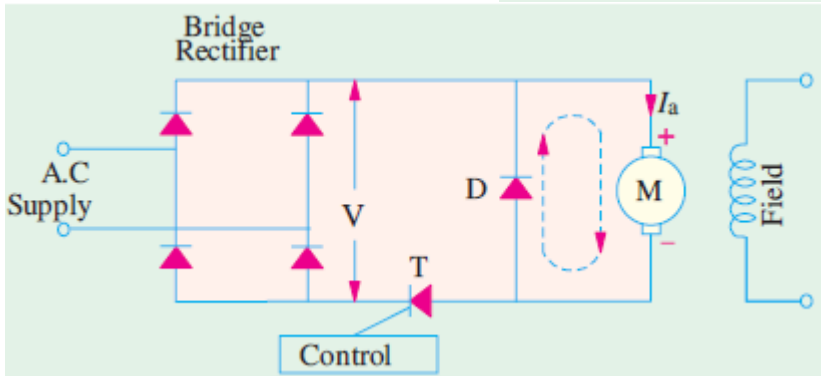
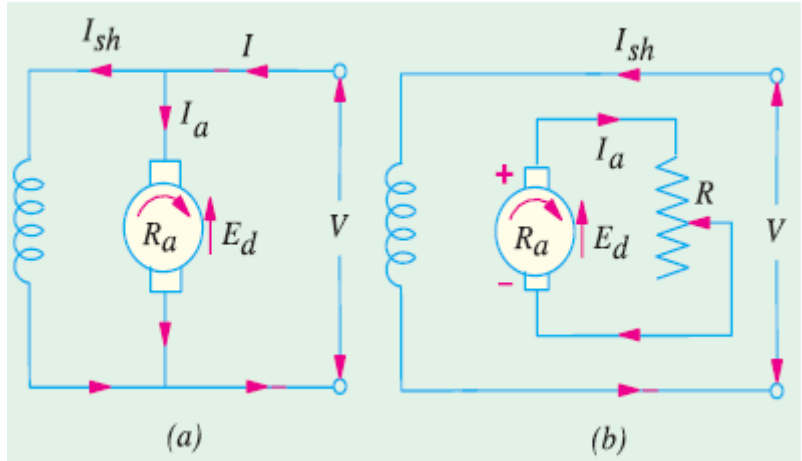
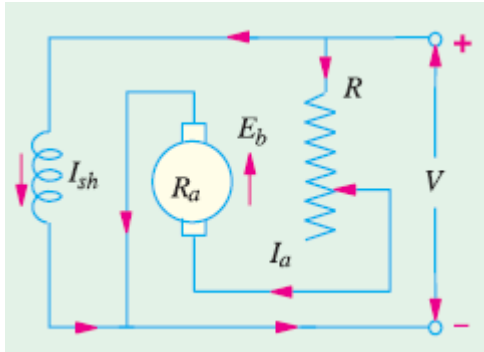
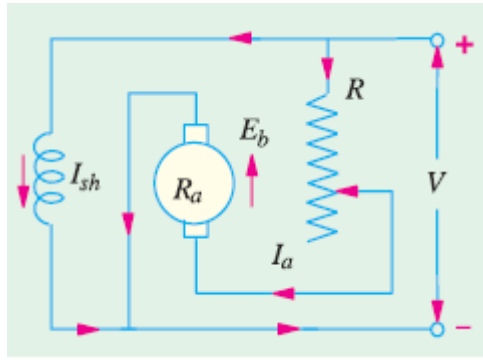
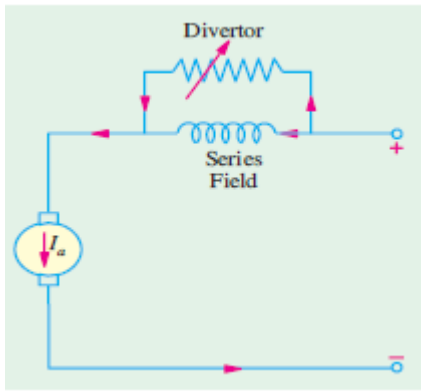
Speed-control Rheostats

whereas the outside field lever moves over the upper ones on the field rheostat. Only the outside field arm is provided with an operating handle. While starting the motor, the two arms are moved together, but field lever is electrically inoperative because the field current flows directly from the starting arm through the brass arc to HOLD-ON coil and finally to the shunt field winding. At the end of the starting period, the starting arm is attracted and held in FULL-ON position by the HOLD-ON coil, and the contact between the starting arm and brass arc is broken thus forcing field current to pass through the field rheostat. The

field lever can be moved back to increase the motor speed. It will be seen that now the upper row of contacts is operative because starting arm no longer touches the brass arc.

When motor is stopped by opening the main switch, the starting arm is released and on its way back it strikes the field lever so that both arms are returned simultaneously to OFF position.

2.c) Explain the starting of DC motor?



### 30.19. Necessity of a Starter

It has been shown in Art 29.3 that the current drawn by a motor armature is given by the relation

$$I_a = (V - E_b)/R_a$$

where  $V$  is the supply voltage,  $E_b$  the back e.m.f. and  $R_a$  the armature resistance.

When the motor is at rest, there is, as yet, obviously no back e.m.f. developed in the armature. If, now, full supply voltage is applied across the stationary armature, it will draw a very large current because armature resistance is relatively small. Consider the case of a 440-V, 5 H.P. (3.73 kW) motor having a cold armature resistance of  $0.25 \Omega$  and a full-load current of 50 A. If this motor is started from the line directly, it will draw a starting current of  $440/0.25 = 1760$  A which is  $1760/50 = 35.2$  times its full-load current. This excessive current will blow out the fuses and, prior to that, it will damage the commutator and brushes etc. To avoid this happening, a resistance is introduced in series with the armature (for the duration of starting period only, say 5 to 10 seconds) which limits the starting current to a safe value. The starting resistance is gradually cut out as the motor gains speed and develops the back e.m.f. which then regulates its speed.

Very small motors may, however, be started from rest by connecting them directly to the supply lines. It does not result in any harm to the motor for the following reasons :

1. Such motors have a relatively higher armature resistance than large motors, hence their starting current is not so high.
2. Being small, they have low moment of inertia, hence they speed up quickly.
3. The momentary large starting current taken by them is not sufficient to produce a large disturbance in the voltage regulation of the supply lines.

In Fig. 30.37 the resistance  $R$  used for starting a shunt motor is shown. It will be seen that the starting resistance  $R$  is in series with the **armature** and not with the **motor** as a whole. The field winding is connected directly across the lines, hence shunt field current is independent of the



Fig. 30.36

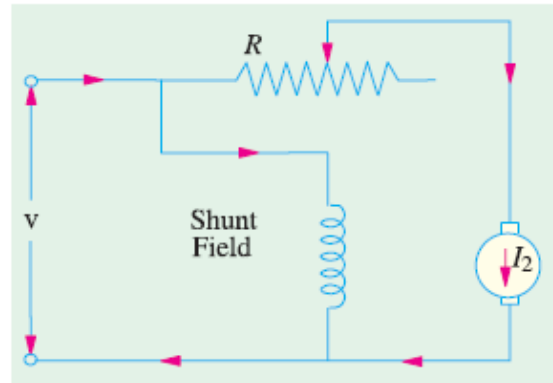
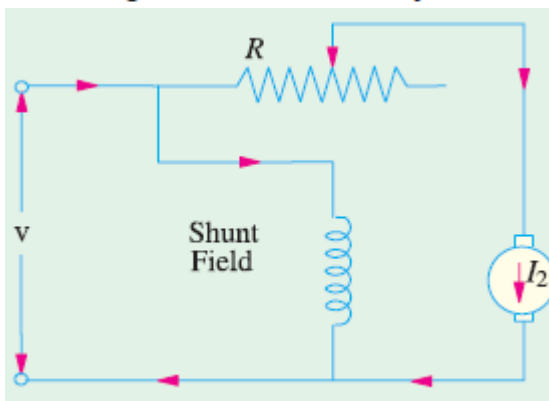


Fig. 30.37

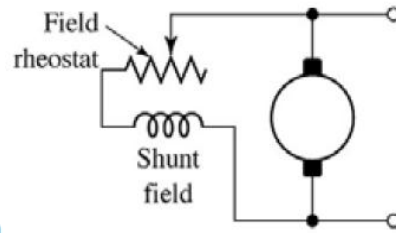


2.d) A 120 V shunt DC motor has the following parameters  $R_a=0.4$  ,  $R_f=120$  and rotational losses are 240 watt on full load the line current is 19.5A and the motor runs at 1200rpm

### Determine

i-Draw the equivalent circuit?

ii- Determine: the armature current, the field current, shaft speed, mechanical power developed, developed torque, and efficiency?



$$T_{\text{mech}} = K_a \Phi_d i_a \quad e_a = K_a \Phi_d \omega_m$$

$$I_f = 120/120 = 1\text{A}, I_a = 19.5 - 1 = 18.5\text{A}, E_a = V_a - I_a R_a = 120 - 18.5 * 0.4 = 112.6\text{V}$$

$$P_{\text{inp}} = 120 * 19.5 = 2340\text{watt}, P_{\text{dev}} = 112.6 * 18.5 = 2083.1\text{W},$$

$$P_{\text{out}} = P_{\text{dev}} - \text{losses} = 2083 - 240 = 1843\text{watt} \quad \eta = 1843/2340 = 79\%$$

2.e) A separately excited DC generator has an open circuit terminal voltage of 144V.

When loaded by resistive load the voltage across the load is 120V. The armature resistance is 0.52  $\Omega$  and the field supply voltage is 220V and field resistance is 147  $\Omega$ .

**Determine** (i) the generated emf, (ii) armature current and field current (iii) the efficiency?

$$E_a = 144\text{V}, I_f = 220/147 = 1.5\text{A}, E_a = V_L + I_a R_a, I_a = (144 - 120)/0.52 = 46.15\text{A},$$

$$P_{\text{inp}} = 220 * 1.5 + 46.15 * 144 = 6975.6\text{W}, P_{\text{out}} = 120 * 46.15 = 5538\text{W}, \eta = 5538/6975.6 = 79.4\%$$

### **Question (3)**

**[15] Points**

3.a) Explain the construction of operation of a 3-phase Induction Motor?

● Main features of synchronous machines:

1-A synchronous machine is an ac machine whose speed under steady-state conditions is proportional to the frequency of the AC current in its armature winding: on the stator.

2-The rotor (field windings dc power supplied by the excitation system ) along with the magnetic field created by the dc field current on the rotor, rotates at the same speed as, or in synchronism with, the rotating magnetic field produced by the armature currents, and a steady torque results.

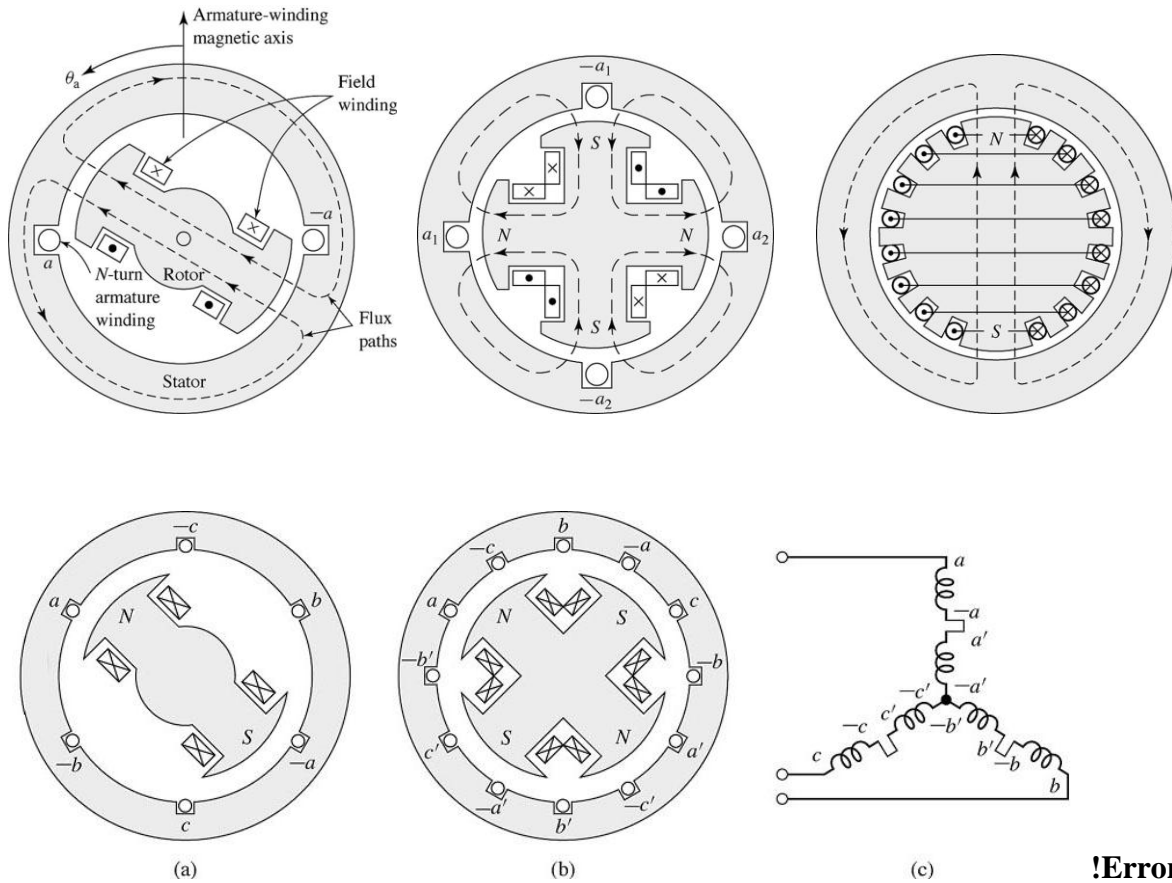


Figure 4.12 Schematic views of three-phase generators:  
 (a) two-pole, (b) four-pole, and (c) Y connection of the windings.

**Construction:**

- 1-Cylindrical rotor: for two- and four-pole turbine generators.
  - 2-Salient-pole rotor: for multipolar, slow-speed, hydroelectric generators
- Frequency determined by the speed of its mechanical drive (or prime mover).

The amplitude of the generated voltage is proportional to the frequency and the field current.

$$(4.45) \lambda_a = k_w N_{ph} \Phi_p \cos\left(\left(\frac{\text{poles}}{2}\right) \omega_m t\right)$$

$$= k_w N_{ph} \Phi_p \cos \omega_{me} t$$

$$(4.46) \omega_{me} = \left(\frac{\text{poles}}{2}\right) \omega_m$$

$$(4.47) e_a = \frac{d\lambda_a}{dt} = k_w N_{ph} \frac{d\Phi_p}{dt} \cos \omega_{me} t - \omega_{me} k_w N_{ph} \Phi_p \sin \omega_{me} t$$

$$(4.48) e_a = -\omega_{me} k_w N_{ph} \Phi_p \sin \omega_{me} t$$

$$(4.49) E_{\max} = \omega_{me} k_w N_{ph} \Phi_p = 2\pi f_{me} k_w N_{ph} \Phi_p$$

$$(4.50) E_{\text{rms}} = \frac{2\pi}{\sqrt{2}} f_{me} k_w N_{ph} \Phi_p = \sqrt{2} \pi f_{me} k_w N_{ph} \Phi_p$$

Torque equation:

$$(5.1) T = -\frac{\pi}{2} \left(\frac{\text{poles}}{2}\right)^2 \Phi_R F_f \sin \delta_{RF}$$



where

= resultant air-gap flux per pole  $\Phi_R$

= mmf of the dc field winding  $F_f$

= electric phase angle between magnetic axes of  $\Phi_R$  and  $F_f$   $\delta_{RF}$

- The minus sign indicates that the electromechanical torque acts in the direction to bring the interacting fields into alignment.
- In a generator, the prime-mover torque acts in the direction of rotation of the rotor, and the electromechanical torque opposes rotation. The rotor mmf wave leads the resultant air-gap flux.
- In a motor, the electromechanical torque is in the direction of rotation, in opposition to the retarding torque of the mechanical load on the shaft.
- Torque-angle curve: Fig. 5.1.

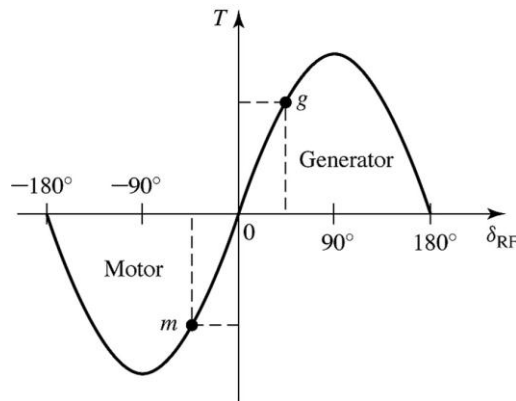


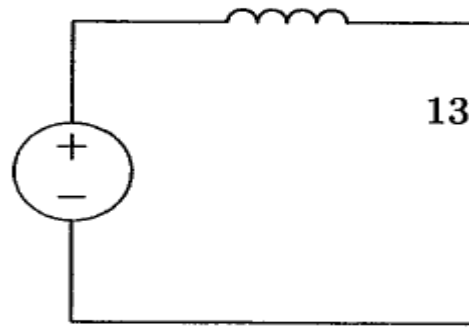
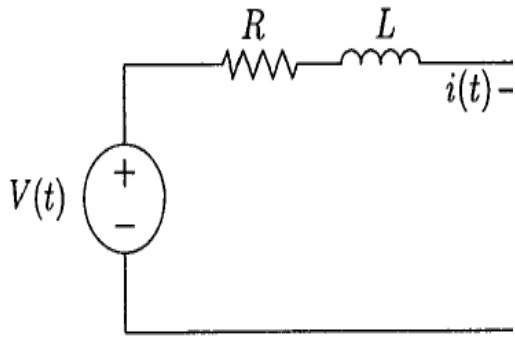
Figure 5.1 Torque-angle characteristics.

- An increase in prime-mover torque will result in a corresponding increase in the torque angle.
- $T = T_{\max}$  : pull-out torque at  $\delta_{RF} = 90^\circ$ . Any further increase in prime-mover torque cannot be balanced by a corresponding increase in synchronous electromechanical torque, with the result that synchronism will no longer be maintained and the rotor will speed up.  $\Rightarrow$  loss of synchronism, pulling out of step.

**3.b)** - A three phase 460V, 60Hz, 6 poles, wye-connected cylindrical rotor synchronous motor has a synchronous reactance of 2  $\Omega$ /phase.  $R_s$  is negligible and  $I_s=22A$ /phase and unity p.f.

i-Draw the equivalent circuit? ii-Find the rotor speed and torque angle?

iii-Find the  $P_{out}$  and the maximum torque?



$$n_r = n_s = 120 \cdot 60 / 6 = 1200 \text{ rpm}, \quad \omega_r = \omega_s = 1200 \cdot \pi / 30 = 125.7 \text{ rad/s}, \quad 460 / \sqrt{3} = 265.6 \angle 0$$

$$V_f = V_t - jI_a X_s = 265.6 \angle 0 - j22 \cdot 2 = 265.6 - j44 = 268.2 \angle -9.4^\circ$$

Torque angle  $\delta = -9.4^\circ$ ,

$$P_{\text{dev}} = \frac{3 * V_{\text{ph}} * V_f * \sin \delta}{X_s} = \frac{3 * 265.6 * 268.2 * \sin 9.4}{2} = 17451.5 \text{ W}$$

$$T_{\text{max}} = \frac{3 * V_{\text{ph}} * V_f}{\omega_s X_s} = 841 \text{ Nm}$$