Benha University
Faculty of Engineering
Mechanical Engineering Dep.

Thermodynamics II
2nd year 2016
Final-exam (code: M 1222)

Time: Three Hours   *(attempt all questions)*   *(assume any missing data)*

Question1   (15 points)

1-1) What is the difference between Stirling, Ericson and Carnot cycles?

They differ from the Carnot cycle in that the two isentropic processes are replaced by two constant-volume regeneration processes in the Stirling cycle and by two constant-pressure regeneration processes in the Ericsson cycle. Both cycles utilize regeneration, a process during which heat is transferred to a thermal energy storage device (called a regenerator) during one part of the cycle and is transferred back to the working fluid during another part of the cycle.

1-2) A four-cylinder, four-stroke, 2.2-L gasoline engine operates on the Otto cycle with a compression ratio of 10. The air is at 100 kPa and 60°C at the beginning of the compression process, and the maximum pressure in the cycle is 8 MPa. The compression and expansion processes may be modeled as polytropic with a polytropic constant of 1.3. Using constant specific heats at 850 K, determine (a) Show the cycle on P-v and T-s diagrams, (b) the temperature at the end of the expansion process, (c) the net work output and the thermal efficiency, (d) the mean effective pressure, (e) the engine speed for a net power output of 70 kW and comment.

Properties The properties of air at 850 K are c_p = 1.110 kJ/kg·K, c_v = 0.823 kJ/kg·K, R = 0.287 kJ/kg·K, and k = 1.349 (Table A-2b).

Analysis (*a*) Process 1-2: polytropic compression

$$T_2 = T_1 \left(\frac{v_1}{v_2} \right)^{n-1} = (333 \text{ K})(10)^{1.3-1} = 664.4 \text{ K}$$

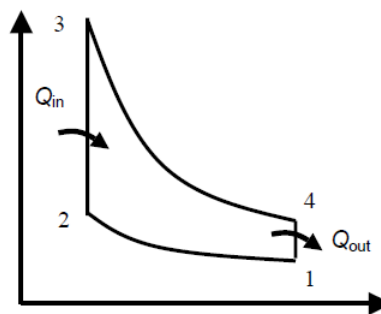$$P_2 = P_1 \left(\frac{v_1}{v_2} \right)^{n} = (100 \text{ kPa})(10)^{1.3} = 1995 \text{ kPa}$$

Process 2-3: constant volume heat addition

$$T_3 = T_2 \left(\frac{P_3}{P_2} \right) = (664.4 \text{ K}) \left(\frac{8000 \text{ kPa}}{1995 \text{ kPa}} \right) = 2664 \text{ K}$$

$$q_{in} = u_3 - u_2 = c_v (T_3 - T_2)$$
$$= (0.823 \text{ kJ/kg} \cdot \text{K})(2664 - 664.4) \text{K} = 1646 \text{ kJ/kg}$$

Process 3-4: polytropic expansion.

$$T_4 = T_3 \left(\frac{v_3}{v_4} \right)^{n-1} = (2664 \text{ K}) \left(\frac{1}{10} \right)^{1.3-1} = \mathbf{1335 \text{ K}}$$

$$P_4 = P_3\left(\frac{v_2}{v_1}\right)^n = (8000 \text{ kPa})\left(\frac{1}{10}\right)^{1.3} = 400.9 \text{ kPa}$$

Process 4-1: constant voume heat rejection.

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1) = (0.823 \text{ kJ/kg} \cdot \text{K})(1335 - 333)\text{K} = 824.8 \text{ kJ/kg}$$

(b) The net work output and the thermal efficiency are

$$w_{net,out} = q_{in} - q_{out} = 1646 - 824.8 = \textbf{820.9 kJ/kg}$$

$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{820.9 \text{ kJ/kg}}{1646 \text{ kJ/kg}} = \textbf{0.499}$$

(c) The mean effective pressure is determined as follows

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(333 \text{ K})}{100 \text{ kPa}} = 0.9557 \text{ m}^3/\text{kg} = v_{max}$$

$$v_{min} = v_2 = \frac{v_{max}}{r}$$

$$\text{MEP} = \frac{w_{net,out}}{v_1 - v_2} = \frac{w_{net,out}}{v_1(1 - 1/r)} = \frac{820.9 \text{ kJ/kg}}{(0.9557 \text{ m}^3/\text{kg})(1 - 1/10)}\left(\frac{\text{kPa} \cdot \text{m}^3}{\text{kJ}}\right) = \textbf{954.3 kPa}$$

(d) The clearance volume and the total volume of the engine at the beginning of compression process (state 1) are

$$r = \frac{V_c + V_d}{V_c} \longrightarrow 10 = \frac{V_c + 0.0022 \text{ m}^3}{V_c} \longrightarrow V_c = 0.0002444 \text{ m}^3$$

$$V_1 = V_c + V_d = 0.0002444 + 0.0022 = 0.002444 \text{ m}^3$$

The total mass contained in the cylinder is

$$m_t = \frac{P_1 V_1}{RT_1} = \frac{(100 \text{ kPa})/0.002444 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(333 \text{ K})} = 0.002558 \text{ kg}$$

The engine speed for a net power output of 70 kW is

$$\dot{n} = 2\frac{\dot{W}_{net}}{m_t w_{net}} = (2 \text{ rev/cycle})\frac{70 \text{ kJ/s}}{(0.002558 \text{ kg})(820.9 \text{ kJ/kg} \cdot \text{cycle})}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \textbf{4001 rev/min}$$

Note that there are two revolutions in one cycle in four-stroke engines.

Question 2   (25 points)

2-1) How can we increase the efficiency of the Brayton cycle?

1- Increasing the turbine inlet (or firing) temperatures
2- Increasing the efficiencies of turbomachinery components
3- Adding modifications to the basic cycle

2-2) The single-stage compression process of an ideal Brayton cycle without regeneration is replaced by a multistage compression process with intercooling between the same pressure limits. As a result of this modification,

(a) The compressor work ***decrease***

(b) The back work ratio ***decrease***

(c) The thermal efficiency ***increase***

2-3) The single-stage expansion process of an ideal Brayton cycle without regeneration is replaced by a multistage expansion process with reheating between the same pressure limits. As a result of this modification,

(a) The turbine work ***increase***

(b) The back work ratio ***increase***

(c) The thermal efficiency ***increase***

2-4) A steam power plant operates on an ideal reheat– regenerative Rankine cycle and has a net power output of 80 MW. Steam enters the high-pressure turbine at 10 MPa and 550°C and leaves at 0.8 MPa. Some steam is extracted at this pressure to heat the feedwater in an open feedwater heater. The rest of the steam is reheated to 500°C and is expanded in the low- pressure turbine to the condenser pressure of 10 kPa. Show the cycle on a T-s diagram with respect to saturation lines, and determine (a) the mass flow rate of steam through the boiler and (b) the thermal efficiency of the cycle.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

Analysis (*a*) From the steam tables (Tables A-4, A-5, and A-6),

$$h_1 = h_{f@10\,kPa} = 191.81\ kJ/kg$$

$$v_1 = v_{f@10\,kPa} = 0.00101\ m^3/kg$$

$$w_{pI,in} = v_1(P_2 - P_1) = (0.00101\ m^3/kg)(800 - 10\ kPa)\left(\frac{1\ kJ}{1\ kPa \cdot m^3}\right)$$

$$= 0.80\ kJ/kg$$

$$h_2 = h_1 + w_{pI,in} = 191.81 + 0.80 = 192.61\ kJ/kg$$

$$P_3 = 0.8\ MPa\ \Big| \begin{array}{l} h_3 = h_{f@0.8\,MPa} = 720.87\ kJ/kg \\ sat.liquid \quad v_3 = v_{f@0.8\,MPa} = 0.001115\ m^3/kg \end{array}$$

$$w_{pII,in} = v_3(P_4 - P_3) = (0.001115\ m^3/kg)(10,000 - 800\ kPa)\left(\frac{1\ kJ}{1\ kPa \cdot m^3}\right)$$

$$= 10.26\ kJ/kg$$

$$h_4 = h_3 + w_{pII,in} = 720.87 + 10.26 = 731.12\ kJ/kg$$

$$P_5 = 10\ MPa\ \Big| \begin{array}{l} h_5 = 3502.0\ kJ/kg \\ T_5 = 550°C \quad s_5 = 6.7585\ kJ/kg\cdot K \end{array}$$

$$P_6 = 0.8\ MPa\ \Big| \begin{array}{l} h_6 = 2812.1\ kJ/kg \\ s_6 = s_5 \end{array}$$

$$P_7 = 0.8\ MPa\ \Big| \begin{array}{l} h_7 = 3481.3\ kJ/kg \\ T_7 = 500°C \quad s_7 = 7.8692\ kJ/kg\cdot K \end{array}$$

$$P_8 = 10\ kPa\ \Big| \begin{array}{l} x_8 = \dfrac{s_8 - s_f}{s_{fg}} = \dfrac{7.8692 - 0.6492}{7.4996} = 0.9627 \\ s_8 = s_7 \\ h_8 = h_f + x_8 h_{fg} = 191.81 + (0.9627)(2392.1) = 2494.7\ kJ/kg \end{array}$$

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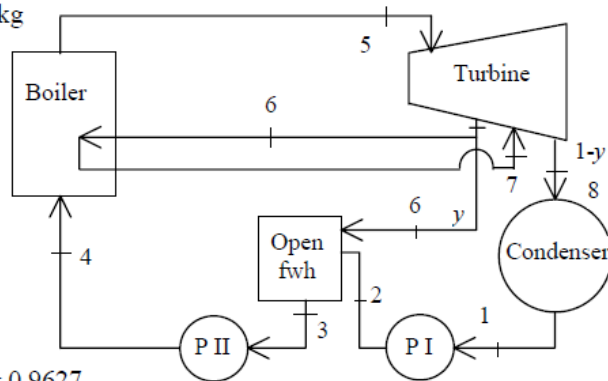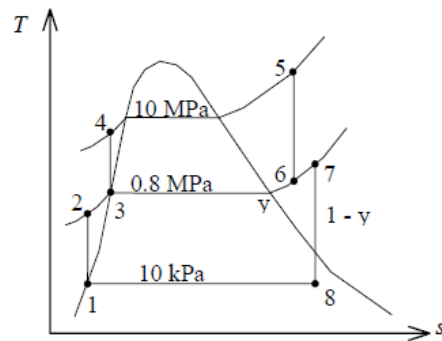
The fraction of steam extracted is determined from the steady-flow energy balance equation applied to the feedwater heaters. Noting that $\dot{Q} \cong \dot{W} \cong \Delta ke \cong \Delta pe \cong 0$,

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{system}^{\nearrow 0 \text{ (steady)}} = 0 \rightarrow \dot{E}_{in} = \dot{E}_{out}$$

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e \longrightarrow \dot{m}_6 h_6 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \longrightarrow y h_6 + (1-y) h_2 = 1(h_3)$$

where y is the fraction of steam extracted from the turbine ($= \dot{m}_6 / \dot{m}_3$). Solving for y,

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{720.87 - 192.61}{2812.1 - 192.61} = 0.2017$$

Then, $\quad q_{in} = (h_5 - h_4) + (1-y)(h_7 - h_6) = (3502.0 - 731.12) + (1 - 0.2017)(3481.3 - 2812.1) = 3305.1 \text{ kJ/kg}$

$\quad q_{out} = (1-y)(h_8 - h_1) = (1 - 0.2017)(2494.7 - 191.81) = 1838.5 \text{ kJ/kg}$

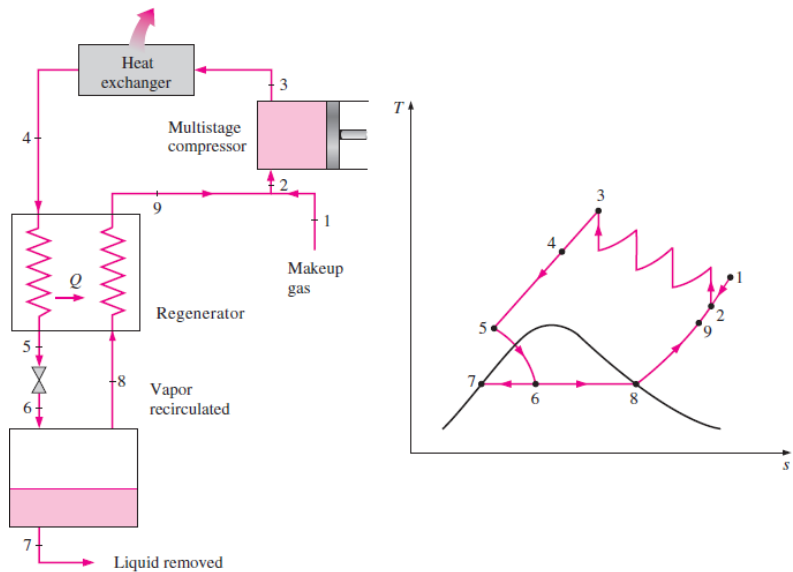$\quad w_{net} = q_{in} - q_{out} = 3305.1 - 1838.5 = 1466.6 \text{ kJ/kg}$

and $\quad \dot{m} = \dfrac{\dot{W}_{net}}{w_{net}} = \dfrac{80,000 \text{ kJ/s}}{1466.1 \text{ kJ/kg}} = \textbf{54.5 kg/s}$

(b) $\quad \eta_{th} = \dfrac{w_{net}}{q_{in}} = \dfrac{1466.1 \text{ kJ/kg}}{3305.1 \text{ kJ/kg}} = \textbf{44.4\%}$

Question 3   (20 points)

3-1) How can we lower the temperature of a gas below its critical-point value for the liquefaction of gases (show your answer schematically and T-s diagram)?

Several cycles, some complex and others simple, are used successfully for the liquefaction of gases. Below we discuss the Linde-Hampson cycle, which is shown schematically and on a *T-s* diagram in figure. Makeup gas is mixed with the uncondensed portion of the gas from the previous cycle, and the mixture at state 2 is compressed by a multistage compressor to state 3. The compression process approaches an isothermal process due to intercooling. The high-pressure gas is cooled in an aftercooler by a cooling medium or by a separate external refrigeration system to state 4.

The gas is further cooled in a regenerative counter-flow heat exchanger by the uncondensed portion of gas from the previous cycle to state 5, and it is throttled to state 6, which is a saturated liquid–vapor mixture state. The liquid (state 7) is collected as the desired product, and the vapor (state 8) is routed through the regenerator to cool the high-pressure gas approaching the throttling valve. Finally, the gas is mixed with fresh makeup gas, and the cycle is repeated.

3-2) Consider a two-stage cascade refrigeration system operating between the pressure limits of 0.8 and 0.14 MPa. Each stage operates on the ideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid. Heat rejection from the lower cycle to the upper cycle takes place in an adiabatic counter flow heat exchanger where both streams enter at about 0.4 MPa. If the mass flow rate of the refrigerant through the upper cycle is 0.24 kg/s, Draw the schematic and T-S diagrams and determine (a) The mass flow rate of the refrigerant through the lower cycle, (b) The rate of heat removal from the refrigerated space and the power input to the compressor, and (c) The coefficient of performance of this cascade refrigerator and COP if this cascade work as a heat pump?

Compare your results with a single stage refrigerator and comment.

Solution
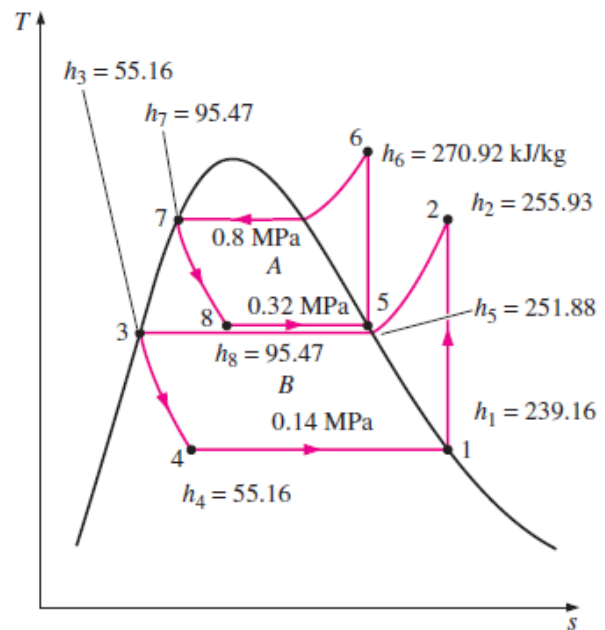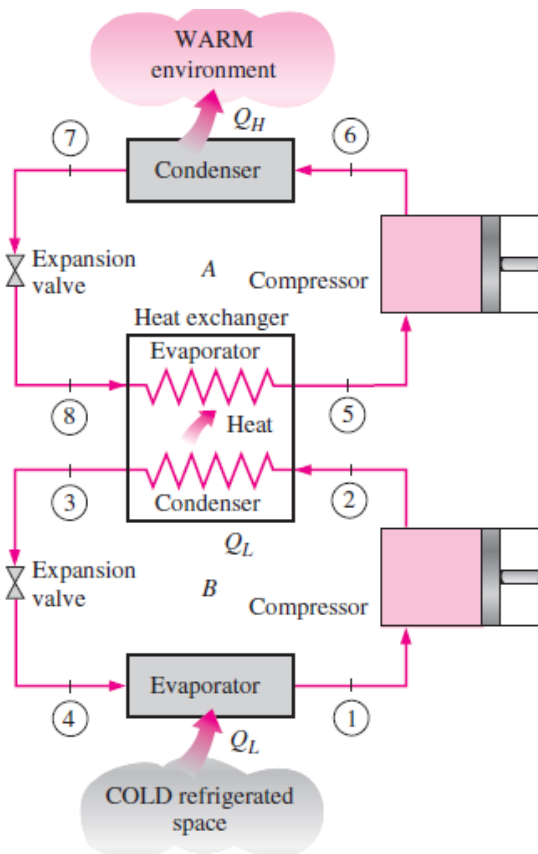
(a) The mass flow rate of the refrigerant through the lower cycle is determined from the steady-flow energy balance on the adiabatic heat exchanger,

$$\dot{E}_{out} = \dot{E}_{in} \longrightarrow \dot{m}_A h_5 + \dot{m}_B h_3 = \dot{m}_A h_8 + \dot{m}_B h_2$$

$$\dot{m}_A(h_5 - h_8) = \dot{m}_B(h_2 - h_3)$$

$$(0.05 \text{ kg/s})[(251.88 - 95.47) \text{ kJ/kg}] = \dot{m}_B[(255.93 - 55.16) \text{ kJ/kg}]$$

$$\dot{m}_B = 0.0390 \text{ kg/s}$$

(b) The rate of heat removal by a cascade cycle is the rate of heat absorption in the evaporator of the lowest stage. The power input to a cascade cycle is the sum of the power inputs to all of the compressors:

$$\dot{Q}_L = \dot{m}_B(h_1 - h_4) = (0.0390 \text{ kg/s})[(239.16 - 55.16) \text{ kJ/kg}] = 7.18 \text{ kW}$$

$$\dot{W}_{in} = \dot{W}_{comp\ I,in} + \dot{W}_{comp\ II,in} = \dot{m}_A(h_6 - h_5) + \dot{m}_B(h_2 - h_1)$$

$$= (0.05 \text{ kg/s})[(270.92 - 251.88) \text{ kJ/kg}]$$

$$+ (0.039 \text{ kg/s})[(255.93 - 239.16) \text{ kJ/kg}]$$

$$= 1.61 \text{ kW}$$

(c) The COP of a refrigeration system is the ratio of the refrigeration rate to the net power input:

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}} = \frac{7.18 \text{ kW}}{1.61 \text{ kW}} = 4.46$$

Discussion This problem was worked out in Example 11–1 for a single-stage refrigeration system. Notice that the COP of the refrigeration system increases from 3.97 to 4.46 as a result of cascading. The COP of the system can be increased even more by increasing the number of cascade stages.

Question 4 (15 points)

4-1) A tank contains 21 kg of dry air and 0.3 kg of water vapor at 30°C and 100 kPa total pressure. Determine (a) the specific humidity, (b) the relative humidity, and (c) the volume of the tank.

Analysis (*a*) The specific humidity can be determined form its definition,

$$\omega = \frac{m_v}{m_a} = \frac{0.3\,\text{kg}}{21\,\text{kg}} = 0.0143\,\text{kg H}_2\text{O/kg dry air}$$

(*b*) The saturation pressure of water at 30°C is

$$P_g = P_{sat\,@\,30°C} = 4.2469\,\text{kPa}$$

Then the relative humidity can be determined from

$$\phi = \frac{\omega P}{(0.622 + \omega)P_g} = \frac{(0.0143)(100\,\text{kPa})}{(0.622 + 0.0143)(4.2469\,\text{kPa})} = 52.9\%$$

(*c*) The volume of the tank can be determined from the ideal gas relation for the dry air,

$$P_v = \phi P_g = (0.529)(4.2469\,\text{kPa}) = 2.245\,\text{kPa}$$

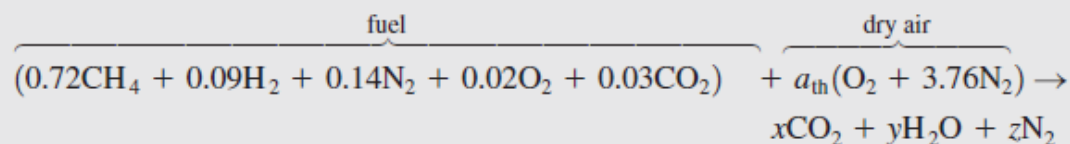$$P_a = P - P_v = 100 - 2.245 = 97.755\,\text{kPa}$$

$$V = \frac{m_a R_a T}{P_a} = \frac{(21\,\text{kg})(0.287\,\text{kJ/kg}\cdot\text{K})(303\,\text{K})}{97.755\,\text{kPa}} = 18.7\,\text{m}^3$$

4-2) A certain natural gas has the following volumetric analysis: 72 percent CH_4, 9 percent H_2, 14 percent N_2, 2 percent O_2, and 3 percent CO_2. This gas is now burned with the stoichiometric amount of air that enters the combustion chamber at 20°C, 1 atm, and 80 percent relative humidity. Assuming complete combustion and a total pressure of 1 atm, determine the dew-point temperature of the products for moist and dry air.

Properties The saturation pressure of water at 20°C is 2.3392 kPa (Table A–4).
Analysis We note that the moisture in the air does not react with anything; it simply shows up as additional H_2O in the products. Therefore, for simplicity, we balance the combustion equation by using dry air and then add the moisture later to both sides of the equation.
Considering 1 kmol of fuel,

$$\underbrace{(0.72CH_4 + 0.09H_2 + 0.14N_2 + 0.02O_2 + 0.03CO_2)}_{\text{fuel}} + \underbrace{a_{th}(O_2 + 3.76N_2)}_{\text{dry air}} \rightarrow$$
$$xCO_2 + yH_2O + zN_2$$

The unknown coefficients in the above equation are determined from mass balances on various elements,

$$\begin{array}{llll}
C: & 0.72 + 0.03 = x & \rightarrow & x = 0.75 \\
H: & 0.72 \times 4 + 0.09 \times 2 = 2y & \rightarrow & y = 1.53 \\
O_2: & 0.02 + 0.03 + a_{th} = x + \dfrac{y}{2} & \rightarrow & a_{th} = 1.465 \\
N_2: & 0.14 + 3.76a_{th} = z & \rightarrow & z = 5.648
\end{array}$$

Next we determine the amount of moisture that accompanies $4.76a_{th} = (4.76)(1.465) = $ 6.97 kmol of dry air. The partial pressure of the moisture in the air is

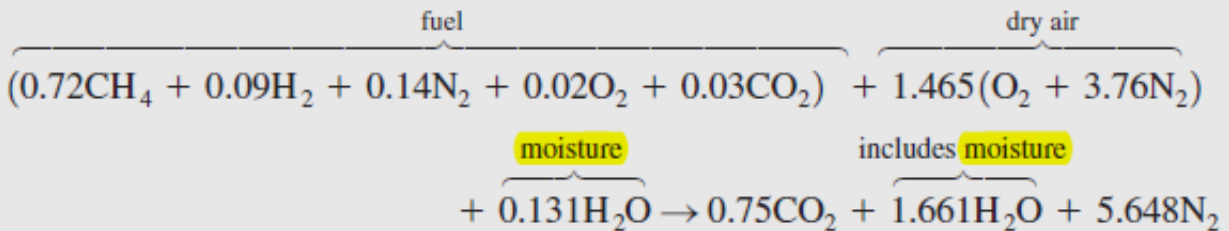$$P_{v,air} = \phi_{air}P_{sat @ 20°C} = (0.80)(2.3392 \text{ kPa}) = 1.871 \text{ kPa}$$

Assuming ideal-gas behavior, the number of moles of the moisture in the air is

$$N_{v,air} = \left(\frac{P_{v,air}}{P_{total}}\right)N_{total} = \left(\frac{1.871 \text{ kPa}}{101.325 \text{ kPa}}\right)(6.97 + N_{v,air})$$

which yields

$$N_{v,air} = 0.131 \text{ kmol}$$

The balanced combustion equation is obtained by substituting the coefficients determined earlier and adding 0.131 kmol of H_2O to both sides of the equation:

$$\underbrace{(0.72CH_4 + 0.09H_2 + 0.14N_2 + 0.02O_2 + 0.03CO_2)}_{\text{fuel}} + \underbrace{1.465(O_2 + 3.76N_2)}_{\text{dry air}}$$

$$+ \underbrace{0.131H_2O}_{\text{moisture}} \rightarrow 0.75CO_2 + \underbrace{1.661H_2O}_{\text{includes moisture}} + 5.648N_2$$

$$P_{v,prod} = \left(\frac{N_{v,prod}}{N_{prod}}\right)P_{prod} = \left(\frac{1.661 \text{ kmol}}{8.059 \text{ kmol}}\right)(101.325 \text{ kPa}) = 20.88 \text{ kPa}$$

$$T_{dp} = T_{sat @ 20.88 \text{ kPa}} = 60.9°C \qquad \text{for dry air } T_{dp} = 59.5°C.$$

With best wishes
Dr. Mohamed Ramadan