جامعة بنها - كلية الهندسة ببنها - قسم الهندسة الكهربية

نموذج الإجابة امتحان مادة هندسة التحكم ك1236 تخلفات مايو يوم الاربعاء الموافق 25-5-2016

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Benha University Benha Faculty of Engineering Control Engineering (E1236)	Time: 3-hours Second Year 25-5-2016 Elect.Eng.Dept.تخلفات

<u>Solve as much as you can questions in two pages</u>

<u>Q1</u> **(20marks)**

a-Write a **mathematical model** represents the physical systems shown in Fig.1, and Fig.2?

b- Draw a block diagram represents the system shown in Fig.2 and using block reduction method to find Y(S)/U(S)?

c-Write the most important features of **a good** control system?

d-Write the most important **advantages and disadvantages** of the **open** loop and the **closed** loop control systems?

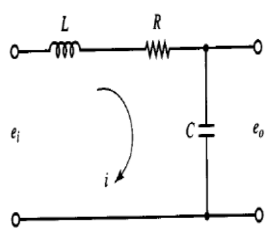


Fig.1

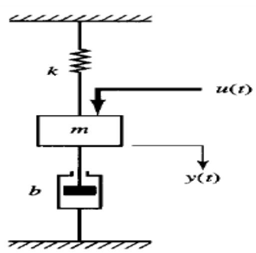


Fig.2

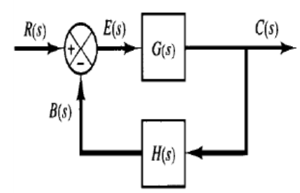


Fig.3

<u>Q2</u> **(20 marks)**

Consider a system shown in Fig. 3 $H(s) = 1$, $G(s) = \frac{K}{S(S+4)}$

a-Find the steady state static **error coefficients**?

b- Find the gain **K** such that the steady state error =0.02?

c- Find and draw the **unit step response** as K=16?

d- Find the **frequency response** and $\mathbf{M_r}$ **and** $\boldsymbol{\omega_r}$ as K=16 and r(t)=2sin ωt ?

P.T.O

Q3 (20 marks)

Consider a control system shown in Fig.3 if

$$G(s)H(s) = \frac{k}{(S+1+j)(S+2)(S+1-j)} = \frac{k}{S^3+4S^2+6S+4}$$

a- Sketch the **complete root locus** for positive values of **K**?
b- Find **K** that makes the complex closed loop poles have a damping ratio =**0.5** and **find the closed loop poles** using **the plot**?
c- Find **K** that makes the complex closed loop poles have a damping ratio =**0.5** and **find the closed loop poles analytically**?
d- Write short MATLAB program to solve a & b?

Q4 (30 marks)

a- Define: gain margin- phase margin?

b- Define: $\boldsymbol{\omega_n}$ **,** $\boldsymbol{\omega_d}$ **,** $\boldsymbol{\omega_r}$ **,** $\boldsymbol{\omega_c}$ **,** $\boldsymbol{\omega_g}$ **,** $\boldsymbol{\omega_p}$ **,** $\boldsymbol{M_r}$**,** $\boldsymbol{\eta}$?

c- Consider a control system shown in Fig.3 if

$$G(s)H(s) = \frac{10}{(S+1+j)(S+2)(S+1-j)} = \frac{10}{S^3+4S^2+6S+4}$$

a- Prove that the gain margin=**6.02 db at 2.45 rad/sec**. and the phase margin= **30.3 degrees at 1.78 rad/sec**.?
b- Sketch the **polar plot?**
c- Sketch the **Bode plot?**
d- Show the gain margin and the phase margin on **the plots**?
e- Write short MATLAB program to solve a , b and C?

Answer

Q1 **(20marks)**

a-Write a **mathematical model** represents the physical systems shown in Fig.1, and Fig.2?

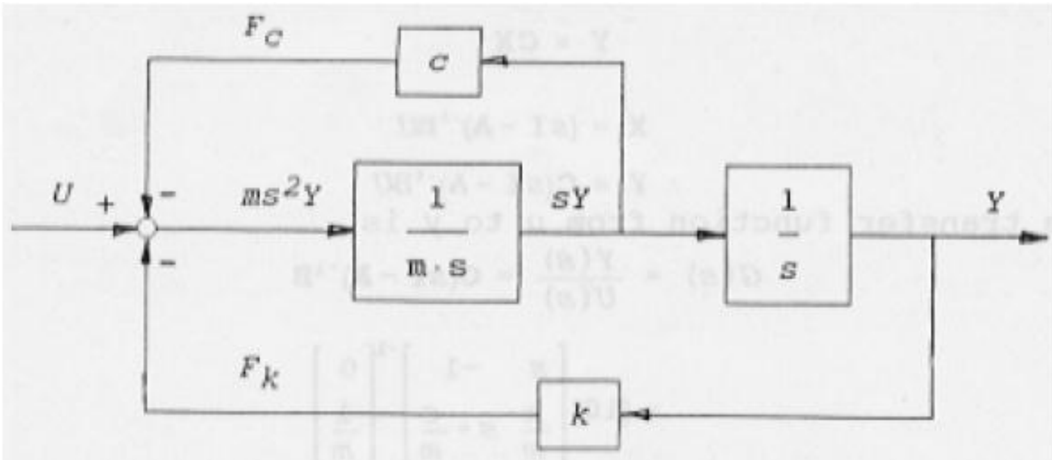
Fig.1

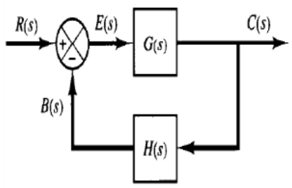
$$\sum_{1}^{n} V_{loop} = 0 \; then \;\; e_i = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}, \;\; i = \frac{dq}{dt}, \qquad e_o = \frac{\int idt}{C} = \frac{q}{c}$$

Fig.2

$$ma = \sum F = m\ddot{y} = U - b\dot{y} - \dot{}Ky,$$

b- Draw block diagram represents the system shown in Fig.2 and using block reduction method to find Y(S)/U(S)?





,K+SC=H(s),G(s)=1/mS^{2,}

$$\text{Closed loop tf} = \frac{C(S)}{R(S)} = \frac{G(s)}{1 + G(S)H(S)} = \frac{1}{mS^2 + Sb + K}$$

c-Write the most important features of **a good** control system?

Most important features of a good control system: are

1-simple construction and operation 2-fast response (speed) 3-less cost

4-very large accuracy (less error) 5-stable

d-Write the most important **advantages and disadvantages** of the **open** loop and the **closed** loop control systems?

Open loop control system

Advantages of open loop	disadvantages of open loop
1-simple construction	1-disturbances cause errors
2- ease of maintenance	2-changes in calibration cause errors
3-less expensive	3-recalibration is necessary
4-no stability problem	
5-convenient when output is hard to measured or economically not feasible	

Closed loop control system

Disadvantages of closed loop	advantages of closed loop
1-complex construction	1-disturbances do not cause errors
2- stability may be a problem	2- has less errors
3-more expensive	3-recalibration is not necessary
	4-the ability to adjust the response

<u>Q2</u> **(20 marks)**

Consider a system shown in Fig. 3 as $\mathrm{H(s)} = 1,\quad \mathrm{G(s)} = \frac{\mathrm{K}}{\mathrm{S(S+4)}}$

a- Find the steady state static **error coefficients**?

It must convert non-unity feedback to unity feedback as

$$\mathrm{K_p} = \lim_0 \mathrm{G(S)} = \lim_{0} \frac{\mathrm{K}}{\mathrm{S(S+4)}} = \frac{\mathrm{K}}{(0)(0+4)} = \infty$$

$$\mathrm{K_v} = \lim_0 \mathrm{SG(S)} = \lim_{0} \frac{\mathrm{K}}{(\mathrm{S}+4)} = \frac{\mathrm{K}}{(0+4)} = 0.25K$$

$$\mathrm{K_a} = \lim_0 \mathrm{S^2G(S)} = \lim_{0} \frac{\mathrm{S\,K}}{(\mathrm{S}+4)} = 0$$

b-Find the gain **K** such that the steady state error =0.02?

$\mathrm{e_{ss}(t)} = \frac{1}{\mathrm{K_v}} = \frac{1}{0.25K} = 0.02,\ K = 200$ Routh test as K=200, system is stable

c-Find and draw the **unit step response** as K=16?

$$\frac{\mathrm{C(S)}}{\mathrm{R(S)}} = \frac{\mathrm{G(s)}}{1 + \mathrm{G(S)H(S)}} = \frac{\omega_n{}^2}{\mathrm{S}^2 + 2\eta\omega_n\mathrm{S} + \omega_n{}^2} = \frac{16}{\mathrm{S}^2 + 4\mathrm{S} + 16}, \omega_n = 4\,\mathrm{rad/sec.},\ \eta = 0.5$$

The step response is under damped Step response of a second order system R(S)=1/s

$$\mathrm{C(S)} = \text{closed loop T.F(S)*R(S)} = \frac{\omega_n{}^2R(S)}{S(\,S^2+2\eta\omega_nS+\omega_n{}^2)} = \frac{16}{S(S^2+4S+16)}$$

$$= \frac{a}{S} + \frac{bs+d}{(S^2+4S+16)}\ partial\ fraction\ ,$$

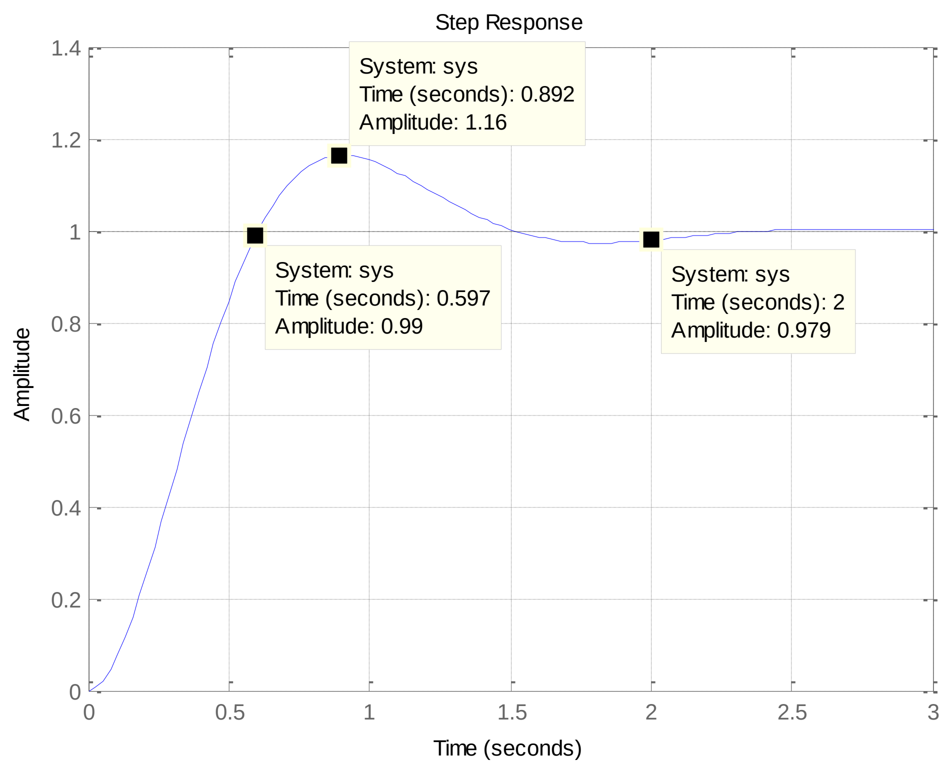
C(t)= inverse Laplace of the product of closed loop t.f.(S) and R(S)=1/s with zero initial conditions C(t)= L^{-1}[(C(S)]=L^{-1}[closed loop t.f.(S)*R(S)] with zero initial conditions]

$$\eta = 0.5, \omega_n = 4rad/sec.\ \omega_d = \omega_n\sqrt{1-\eta^2} = 3.5\,rad/sec\ , cos^{-1}0.5 = pi/3$$

$$C(t) = 1 - \frac{e^{-\eta\ \omega_n t}}{\sqrt{1-\eta^2}} \sin(\ \omega_d t + cos^{-1}\eta) \quad = 1 - 1.155e^{-2t}\sin(1.732t + pi/3)$$

$$M_{p=}e^{\frac{-\eta\pi}{\sqrt{1-\eta^2}}} = 0.163, t_r = \frac{\pi - cos^{-1}\eta}{\omega_d} = \frac{\pi - \frac{pi}{3}}{3.5} = 0.6sec,$$

$$t_p = \frac{\pi}{\omega_d} = 0.9sec., t_s = 4T = \frac{4}{\eta\omega_n} = 2sec.$$



Find the **frequency response** and $\mathbf{M_r}$ **and** $\boldsymbol{\omega_r}$ as K=4 and r(t)=5 sin ωt ?

d-Find the **frequency response** and $\mathbf{M_r}$ **and** $\boldsymbol{\omega_r}$ as K=16 and r(t)=2sin ωt ?

$$\frac{C(S)}{R(S)} = \frac{\mathrm{G(s)}}{1 + G(S)H(S)} = \frac{{\omega_n}^2}{S^2 + 2\eta\omega_n S + {\omega_n}^2} = \frac{16}{S^2 + 4S + 16}$$

$$\omega_n = 4\,\mathrm{rad/sec.},\quad \eta = 0.5, M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{1}{2(0.5)\sqrt{1-(0.5)^2}} = 1.155$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} = 4\sqrt{1-2(0.5)^2} = 2.818 \text{ rad/sec.}$$

the frequency response as r(t)=2sinωt . Steps to find frequency Response:

1- the closed loop transfer function =**T(s)=C(S)/R(S)** =

$$\frac{C(S)}{R(S)} = \frac{G(s)}{1+G(S)H(S)} = \frac{\omega_n{}^2}{S^2+2\eta\omega_n S+\omega_n{}^2} = \frac{16}{S^2+4S+16}$$

2-the closed loop frequency transfer function =

$$\mathbf{T(j\omega)=C(j\omega)/R(j\omega)} = \frac{16}{(j\omega)^2+4(j\omega)+16} = M\llcorner\Phi = Re+j\ imag$$

$$\boldsymbol{M} = \frac{\mathbf{16}}{\sqrt{(\mathbf{16}-\boldsymbol{\omega}^2)^2+\mathbf{16}\boldsymbol{\omega}^2}} \quad , \ \boldsymbol{\Phi} = \mathbf{tan^{-1}}[\mathbf{4}\boldsymbol{\omega}/(\mathbf{16}-\boldsymbol{\omega}^2)]$$

3-As the input =$r(t) = 3sin\omega t$ then

the response = $C(t) = 2Msin(\omega t + \Phi)$

$$= \frac{\mathbf{32}}{\sqrt{(\mathbf{16}-\boldsymbol{\omega}^2)^2+\mathbf{16}\boldsymbol{\omega}^2}}\sin[\omega t + \mathbf{tan^{-1}}[\mathbf{4}\boldsymbol{\omega}/(\mathbf{16}-\boldsymbol{\omega}^2)]$$

Q3 (15 marks)

Consider a control system shown in Fig.3 if

$$G(s)H(s) = \frac{k}{(S+1+j)(S+2)(S+1-j)} = \frac{k}{S^3+4S^2+6S+4}$$

a- Sketch the **complete root locus** for positive values of **K**?

b- Find **K** that makes the complex closed loop poles have a damping ratio =**0.5** and **find the closed loop poles** using **the plot**?

c- Find **K** that makes the complex closed loop poles have a damping ratio =**0.5** and **find the closed loop poles analytically**?

d- Write short MATLAB program to solve a & b?

a-Root locus:

1-the root locus is symmetrical about the real axis in the S-plane

2-the open loop $G(s)H(s) = \frac{k}{(S+1-j)(S+2)(S+1+j)} = \frac{k}{S^3+4S^2+6S+4}$

3-the root locus starts at the pole and ends at the zeros or infinity

4-number of root loci= n=number of poles of the open loop TF =3 at [-1+j,-1-j,-2]

5-number of zeros= m=0

6-number of asymptotes = n-m=3-0=3

8-center of gravity = $A = \frac{\sum poles - \sum zoles}{n-m} = \frac{-1-1-2}{3} = -1.3$point of intersection of asymptotes with real axis=

9-angles of asymptotes are = $\Theta = \frac{\pm 180(2R+1)}{n-m} = \pm 60, \pm 180$

10- Points of crossing the imaginary axis as Routh test

Charct.equa=1+G(S)H(S)=0= $S^3+4S^2+6s+4+K$

S^3	1	6	4+K≥0, [20-K]/ 4≥0 then -4≤K≤20,Kc=20 $4S^2+24=0$, S=±jω=±$\sqrt{6}$ rad/sec
S^2	4	4+K	
S	[24-4-K]/ 4		
S^0	4+K		

11- there is no break points (break away or break in) at

$$-\frac{dK}{dS} = 0 = \frac{d}{dS}\left[\frac{1}{G(S)H(S)}\right] = \frac{d}{dS}[S^3 + 4S^2 + 6s + 4] = 3S^2 + 8s + 6 = 0$$

12- There is no break angles [±180(2R+1)/r] where r=number of branches(poles for break away or zeros for break in) R=0,1,-----no break angles

13-the angle of departures (complex poles) =

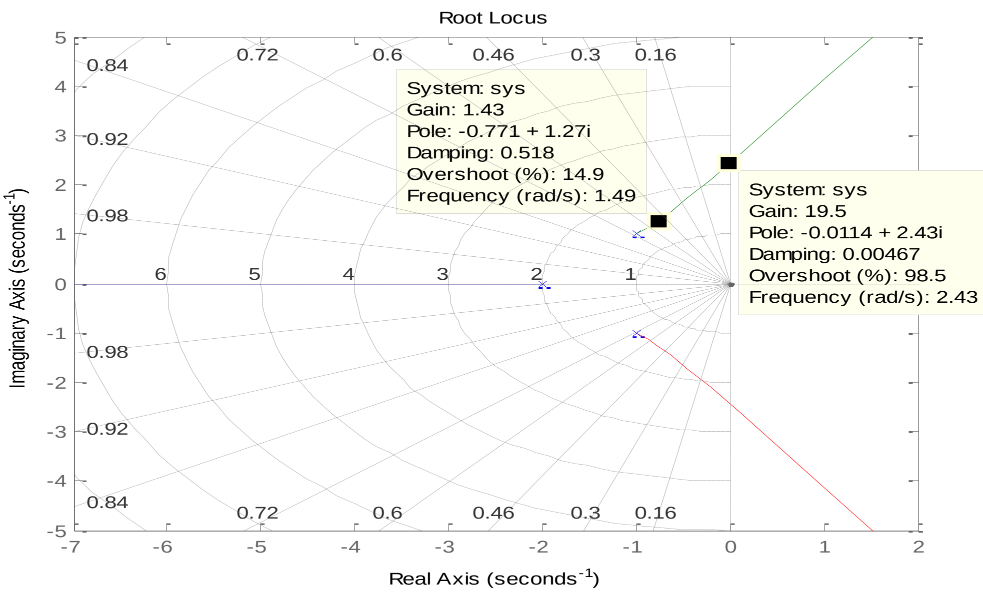
Angle of departure from a complex pole $-$ 180°
$-$ (sum of the angles of vectors to a complex pole in question from other poles)
$+$ (sum of the angles of vectors to a complex pole in question from zeros)

angle of departure=±180-90-45=±45deg

14-angle of arrival (complex zeros) as

Angle of arrival at a complex zero = 180°
$-$ (sum of the angles of vectors to a complex zero in question from other zeros)
$+$ (sum of the angles of vectors to a complex zero in question from poles)

15-sketch the root loci as



16- the damping factor or coefficient ζ is straight line with slope $\Theta = \cos^{-1}\zeta$

with respect to the negative real axis in the S-plane. $\Theta = \cos^{-1} 0.5$ =60deg. at the test point (intersection point) S_d=-0.8±j1.3

$$\text{angle condition} = \sum_{n=1}^{n=3}[\Theta_{zeros} - \Theta_{poles}] = \pm 180(2R+1) = 90 + 54 + 36 = 180 \text{ deg}$$

$$\text{magnitude condition} = \sum_{n=1}^{n=3}\frac{\|\text{poles}\|}{\|\text{zeros}\|} = K = \ * \ \ * \ = 1.5$$

$$\sum_{n=1}^{n=3} open\ loop\ poles = \sum_{n=1}^{n=3} closed\ loop\ poles = \text{constant as } n - m \geq 2$$

$$\sum_{n=1}^{n=3} open\ loop\ poles = -1 - 2 - 1 = -4 = \sum_{n=1}^{n=3} closed\ loop\ poles$$
$$= (-0.8 + j1.3, -0.8 - j1.3, p)$$

$$\text{then } p = -2.4 \text{ i.e. closed loop poles are } [-0.8 \pm j1.3, -2.4\,]$$

19- To find analytically closed loop poles and K as

$(S^2+2\zeta\omega_n S+\omega_n^2)(S+a)$=characteristic equa. for a third order syst.

Solve 1+G(S)H(S)=0= $S^3+4S^2+6s+4+K=(S^2+\omega_n S+\omega_n^2)(S+a)$

$= S^3+(\omega_n+a)S^2+(\omega_n a+\omega_n^2)S+\omega_n^2 a$

$\omega_n + a = 4$, $\omega_n a+\omega_n^2=6$, $\omega_n^2 a=k+4$,then ω_n=1.5rad/se., a=2.5,k=1.74

Prog. >>n=[1];d=[1 4 6 4]; rlocus(n,d), grid

Q4 (30 marks)

a- Define: gain margin- phase margin?

-Gain margin G_m: it is reciprocal of the magnitude of the output frequency response at the **Phase crossover frequency ω_p**

G_m=1/[Real of G(j ω_p)H(j ω_p)]= $1/|G(j\,\omega p)H(j\,\omega p)|$ =K_c/K

G_M=20log G_m db

-Phase margin γ_m: it is the angle of the output frequency response at the **gain crossover** frequency plus 180 degrees.

$\gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180$ deg.

b- Define: ω_n , ω_d , ω_r , ω_c , ω_g , ω_p , M_r, η?

-Natural frequency ω_n rad/sec**:** it is the natural frequency depends on the natural of the system parameters.

- Under damped natural frequency ω_d rad/sec**:** it is the under damped natural frequency depends on the damping coefficient **η** as it is less than one **η** <1.

-Resonant frequency ω_r rad/sec**:** it is the frequency at which the peak value of the output frequency response for a second order is equal to $\boldsymbol{\omega_r = \omega_n\sqrt{1-2\zeta^2}}$

$\omega_r = \omega_n\sqrt{1-2\zeta^2}$, for $0 \le \zeta \le 0.707$

As ζ approaches zero, M_r approaches infinity

$0 < \zeta \le 0.707$, the resonant frequency ω_r is less than the damped natural frequency

-Corner frequency ω_c rad/sec**:** it is the frequency at which the magnitude of the output frequency response is changed sharply. It may be (0, 1, 1/T, **ω_n)**

-Gain crossover frequency ω_g: it is the frequency at which the magnitude of the output frequency response is equal to one or zero decibel.

$|G(j\,\boldsymbol{\omega g})H(j\,\boldsymbol{\omega g})| = 1$ or $|G(j\,\boldsymbol{\omega g})H(j\,\boldsymbol{\omega g})| = 0db$

-Phase crossover frequency ω_p: it is the frequency at which the phase of the output frequency response is equal to (-180) degrees.

Imag. [G(j ω_p)H(j ω_p)]=0 or $\angle G(j\,\boldsymbol{\omega p})H(j\,\boldsymbol{\omega p}) = -180$deg.

-Maximum resonant magnitude M_r: it is the peak value of the output frequency response for a second order system $\boldsymbol{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad M_r = |G(j\omega)|_{max} = |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

-damping coefficient η it depends on the natural of the system parameters. For second order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Values of η	**System stability**	**Step-response**
0> η	System is unstable	undefined
η =0	System is critically stable	oscillatory
0< η <1	System is stable	Under-damped
0< η =1	System is stable	Critically damped
0<η >1	System is stable	Over damped

c-Consider a control system shown in Fig.1 if

$$\mathrm{G(s)H(s)} = \frac{10}{(\mathrm{S}+1+\mathrm{j})(\mathrm{S}+2)(\mathrm{S}+1-\mathrm{j})} = \frac{10}{\mathrm{S}^3+4\mathrm{S}^2+6\mathrm{S}+4}$$

a- Prove that the gain margin=**6.02 db at 2.45 rad/sec**. and the phase margin= **30.3 degrees at 1.78 rad/sec**.?
b- Sketch the **polar plot?**
c- Sketch the **Bode plot?**
d- Show the gain margin and the phase margin on **the plots**?
e- Write short MATLAB program to solve a , b and C?
1- the open loop TF=G(s) H(s)= **G(S) H(S)**

$$\mathrm{G(s)H(s)} = \frac{10}{(\mathrm{S}+1+\mathrm{j})(\mathrm{S}+2)(\mathrm{S}+1-\mathrm{j})} = \frac{10}{\mathrm{S}^3+4\mathrm{S}^2+6\mathrm{S}+4}$$

2- Find the freq.open loop TF=
G(jω)H(jω)=$\frac{10}{\mathrm{S}^3+4\mathrm{S}^2+6\mathrm{S}+4} = \mathrm{Me}^{j\Phi} = M\llcorner\Phi$ =Re+j imag

$$\boldsymbol{M = \frac{10}{\sqrt{(4-4\omega^2)^2+(6\omega-\omega^3)^2}}} \quad , \boldsymbol{\Phi = -\tan^{-1}((6\omega-\omega^3)/(4-4\omega^2))}$$

$$M = \frac{\mathbf{10}}{\sqrt{\mathbf{(4-4\omega^2)^2+(6\omega-\omega^3)^2}}} = \frac{\mathbf{10}}{\sqrt{\mathbf{(4-4\omega^2)^2+(6\omega-\omega^3)^2}}} = 1$$

$$M = \frac{10}{\sqrt{(4-4(1.78)^2)^2+(6(1.78)-(1.78)^3)^2}} = 1, \text{ then } \omega_g = 1.78 rad/sec.$$

$\Phi = -\tan^{-1}((6\omega-\omega^3)/(4-4\omega^2)) = -\tan^{-1}((6 * 2.45 - 2.45^3)/(4 - 4 * 2.45^2)) = -180$ deg.

then $\omega_p = 2.45 rad/sec.$

$$M = \frac{10}{\sqrt{(4-4(2.45)^2)^2+(6(2.45)-(2.45)^3)^2}} = 0.5, \text{then } G_M = 20log\frac{1}{0.5} = 6.02db$$

$\Phi = -\tan^{-1}((6\omega-\omega^3)/(4-4\omega^2)) = -\tan^{-1}((6 * 1.78 - 1.78^3)/(4 - 4 * 1.78^2)) = -149.7$deg. .

$\gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180 \text{ deg.} = 180 - 149.7 = 30.3deg.$

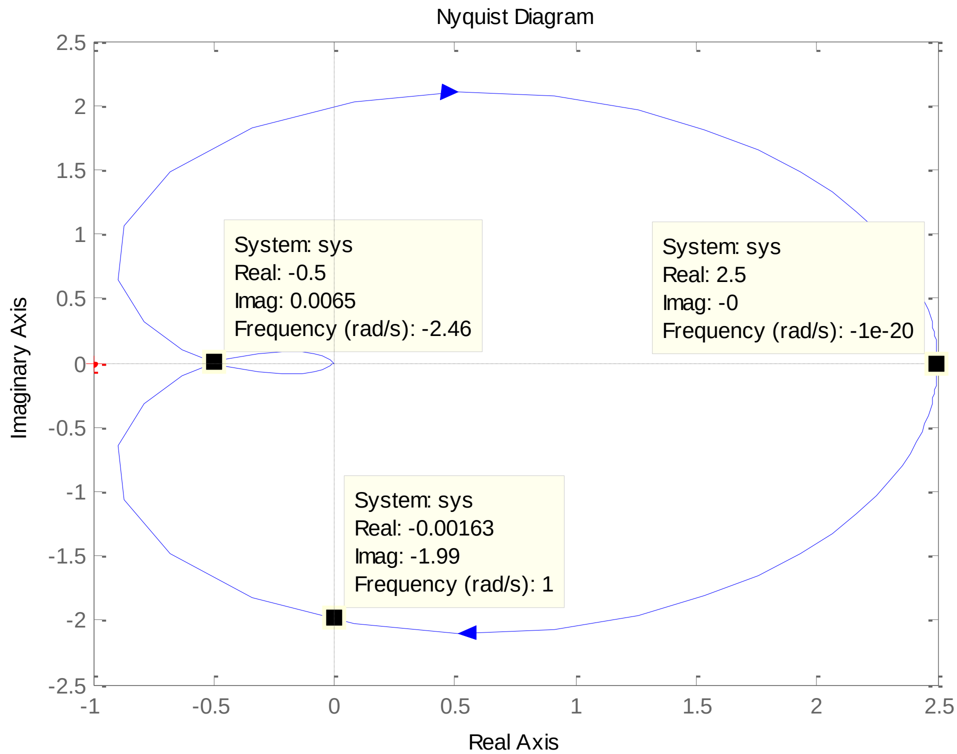
3- Find the table

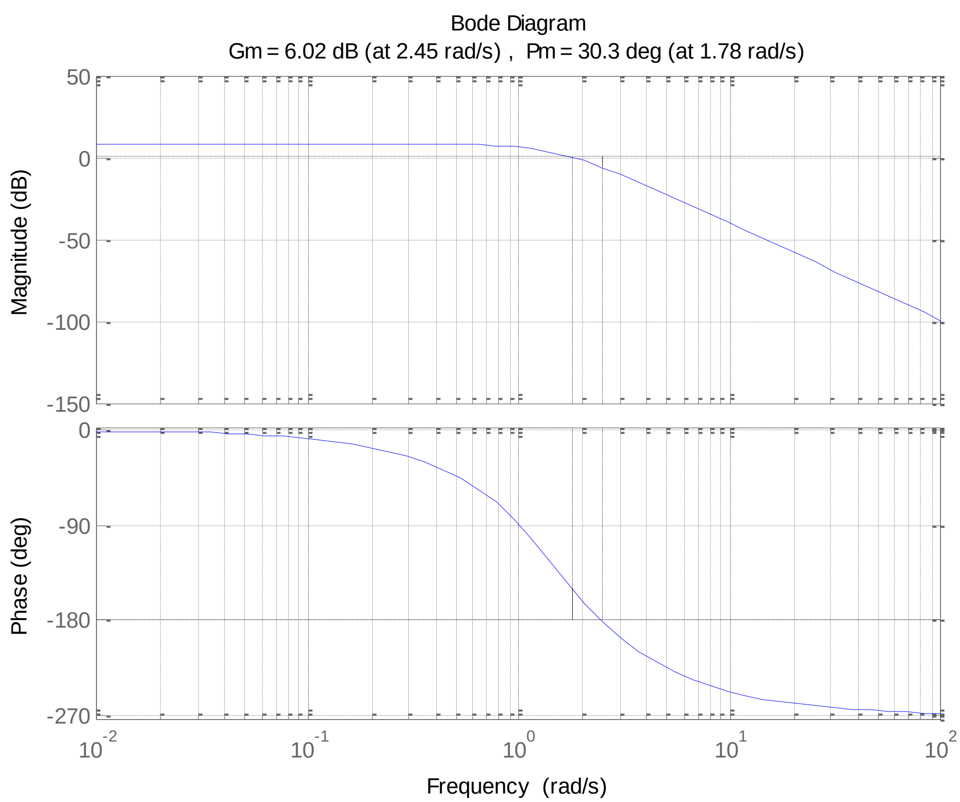
ω	0	0.1	1	1.78	2.45	5	10	∞
Φ	0			-150	-180	-		-270
M	2.5		2	1	0.5			0
20logM	8		6.02-	0	6.02	-	-	0
Real **G(jω)H(jω)**	2.5		0		-0.5			0
Imag **G(jω)H(jω)**	0		-2		0			0

4- Plot the vector on the **jω − plane** where Φ in degrees as a straight line and determine M on this line
5- Plot the locus of the vector as points from the table
6- Find the gain and the phase margins from the plot

<u>Prog.</u> >>n=[10]; d=[1 4 6 4];

>> nyquist(n,d) >> margin(n,d) >> nichols(n,d)


Nyquist Diagram
System: sys
Real: -0.5
Imag: 0.0065
Frequency (rad/s): -2.46
System: sys
Real: 2.5
Imag: -0
Frequency (rad/s): -1e-20
System: sys
Real: -0.00163
Imag: -1.99
Frequency (rad/s): 1
Imaginary Axis
Real Axis


Bode Diagram
Gm = 6.02 dB (at 2.45 rad/s) , Pm = 30.3 deg (at 1.78 rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)