Benha University
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Department of Mechanical Eng.
Subject : Turbo machine

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Model Answer of the Final Exam

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المادة : ألات تربينية

1-a) 1-a) For the turbine, the hydraulic efficiency is defined as

$$\eta = \frac{\text{power delivered to runner}}{\text{power available to runner}}$$

$$\eta = P/\rho/\rho g$$

Then substituting for P and rearranging gives

$$\eta = \overline{P}(ND^3/Q)(N^2D^2/gH)$$

$$\eta = \overline{P}/\varphi\varphi$$

For a pump

$$\eta = \varphi\psi/\overline{P}$$

b) First of all, let us assume that dynamic similarity exists between the two pumps. Equating the flow coefficients,

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \quad \text{or} \quad \frac{2.5}{2010 \times (0.125)^3} = \frac{Q_2}{2210 \times (0.104)^3}$$

Solving the above equation, the volume flow rate of the second pump is

$$Q_2 = \frac{2.5 \times 2210 \times (0.104)^3}{2010 \times (0.125)^3} = 1.58 \text{ m}^3/\text{s}$$

Now, equating head coefficients for both cases gives

$$gH_1/N_1^2D_1^2 = gH_2/N_2^2D_2^2$$

Substituting the given values,

$$\frac{9.81 \times 14}{(2010 \times 125)^2} = \frac{9.81 \times H_2}{(2210 \times 104)^2}$$

Therefore, $H_2 = 11.72$ m of water.

c)

Specific speed can be expressed in this form

$$N_s = N\sqrt{Q}/(gH)^{3/4} = N\sqrt{P}/[\rho^{1/2}(gH)^{5/4}]$$

The specific speed parameter expressing the variation of all the variables N, Q and H or N,P and H, which cause similar flows in turbomachines that are geometrically similar.

Use the specific speed for turbine

$$N_s = N\sqrt{P}/[\rho^{1/2}(gH)^{5/4}]$$

For N_s=450 and the given values $\sqrt{p} = \frac{450\sqrt{1000}\times(12)^{5/4}}{150} = 2118.8$

$p = 4489475.7$ Watt

Number of turbines = 30000000/4489475.7 = 6.68

Number of turbines = 7

2-a) A typical pumping system is shown in Figure 1 where

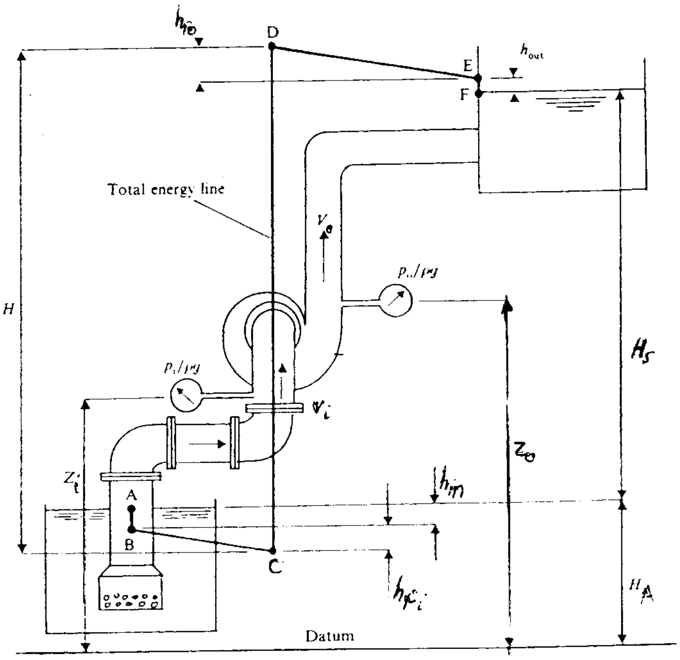


Figure 1. Pumping system

2-b)

2-c) Solution:

Head developed in each stage is

$$H^{3/4} = \frac{N\sqrt{Q}}{N_s} = \frac{1445\sqrt{0.0352}}{14}$$

H = 51:93m

Total head required= 845m (of water)

Number of stages needed =845/51.93 =16

Number of stages in each pump = 8

Impeller speed at tip is

$$U_2 = 0.96(2gH)^{0.5}$$
$$= 0.96[2\times 9.81\times 51.93]^{0.5}$$
$$= 30.6 \text{ m/s}$$

Impeller diameter at tip, $D_2 = \pi\times 60\times 30.6\times 1445$. But

$$U_2 = \frac{\pi D_2 N}{60}$$

or

$$D_2 = \frac{U_2\times 60}{\pi\times 1445} = \frac{30.6\times 60}{\pi\times 1445} = 0.4043 \text{ m} \quad \text{or} \quad 40.43 \text{ cm}.$$

2-c)

Exercise 2.9 (a) Figure 2.39 shows the head-flow and efficiency characteristics plotted for the speed of 750 rpm. Since water is being transferred between reservoirs of the same water level, then from Eq. (2.53),

$$\text{System resistance} = KQ^2$$

Solving for K at the point given:

$$K = 35/25^2 = 0.056$$

Therefore the system head loss at the different flow rates may be calculated:

Q (m^3/min)	0	7	14	21	28	35	42	49	56
System loss (m)	0	2.74	11	24.7	43.9	68.6	–	–	–

The system resistance curve is now drawn (note that it passes through zero) and the head and flow read off at point A. The corresponding efficiency is read off at point B.

At the operating point

$$\underline{Q = 26\ \text{m}^3/\text{min}}$$

$$\underline{H = 38.3\ \text{m}}$$

$$\underline{\eta = 81\ \text{per cent}}$$

(b) Sum of the head losses and static head is given by Eq. (2.54):

$$H = H_s + KQ^2$$

The head losses may be written as

$$\text{Head losses} = \frac{4flv^2}{2gd} + k_{\text{exit}}\frac{v^2}{2g} + k_{\text{entry}}\frac{v^2}{2g}$$

$$= \left(\frac{4 \times 93 \times 0.004}{0.45} + 1 + 0.5\right)\frac{v^2}{2g}$$

$$= (3.31 + 1 + 0.5)\frac{v^2}{2g}$$

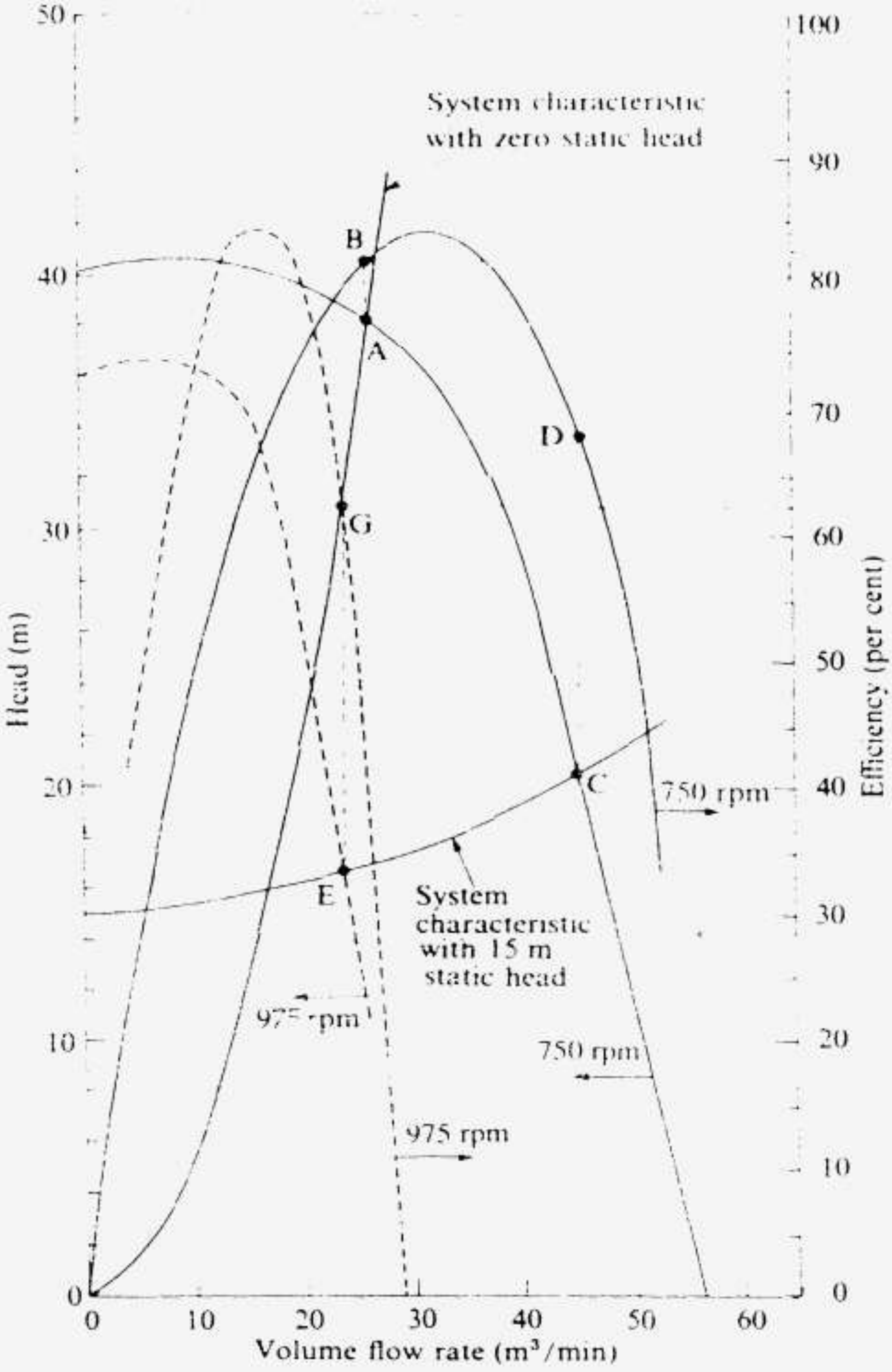


Figure 2.39 Pump characteristics at 750 and 900 rpm

Figure 2

Now

$$v = Q/A$$

and subsituting for v

$$\text{Head losses} = \left[\frac{4.81 \times 4^2}{2 \times 9.81(\pi \times 0.45^2)^2} \right] Q^2$$

$$= 9.69Q^2 \text{ m}$$

Including the static lift

$$\text{System loss} = 15 + 9.69Q^2 \text{ m} \qquad (Q \text{ in m}^3/\text{s})$$

The head loss is now determined for the various flow rates.

Q(m³/min)	0	7	14	21	28	35	42	49
H(m)	15	15.13	15.52	16.18	17.11	18.3	19.75	21.47

The new system resistance curve is drawn noting that it begins at $H = 15$ m. The operating point is at point C and the corresponding efficiency at point D.

At the operating point

$$Q = 45 \text{ m}^3/\text{min}$$

$$H = 20.4 \text{ m}$$

$$\eta = 68.4 \text{ per cent}$$

$$\text{Power absorbed} = \frac{\rho g Q H}{\eta}$$

$$= \frac{10^3 \times 9.81 \times 45 \times 20.4}{0.684 \times 60}$$

$$= 219.4 \text{ kW}$$

(c) Since we have static lift, it is necessary to construct part of the characteristic at the new speed of 900 rpm. The corresponding points for the new impeller and the new speed are found from Eq. (1.6):

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \quad \text{and} \quad \frac{H_1}{(N_1 D_1)^2} = \frac{H_2}{(N_2 D_2)^2}$$

whence

$$Q_2 = Q_1 \left(\frac{975}{750}\right)\left(\frac{0.51}{0.7}\right)^3$$

$$= 0.503Q_1$$

$$H_2 = H_1 \left(\frac{975 \times 0.51}{750 \times 0.7}\right)^2$$

$$= 0.9H_1$$

Q_1	0	7	14	21	28	35	42	49	56
Q_2	0	3.5	7.1	10.6	14.2	17.7	21.3	24.8	28.3
H_1	40	40.6	40.4	39.3	38	33.6	25.6	14.5	0
H	36	36.5	36.4	35.4	34.2	30.24	23.0	13.1	0

The new characteristic is drawn and also the efficiency curve by moving the corresponding values of efficiency horizontally across. The operating point is at E and the corresponding efficiency at G.

At the operating point

$$Q = 23.75\ \text{m}^3/\text{min}$$

$$H = 16.5\ \text{m}$$

$$\eta = 62.4\ \text{per cent}$$

$$\text{Power absorbed} = \frac{23.75 \times 16.5 \times 10^3 \times 9.81}{0.624 \times 60}$$

$$= 102.7\ \text{kW}$$

3-a

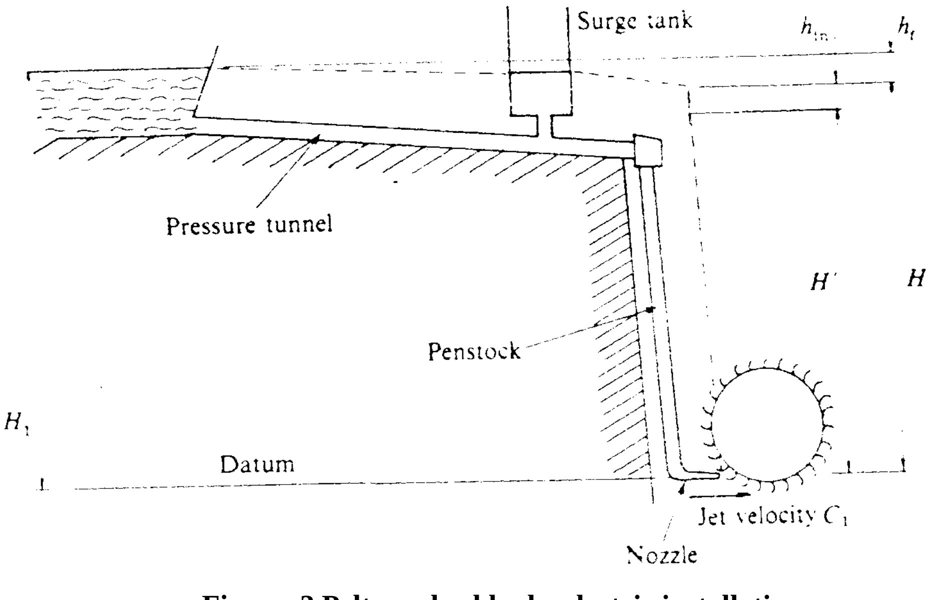


Figure. 3 Pelton wheel hydroelectric installation

$$\text{Pipeline transmission efficiency} = \frac{\text{Energy at end of pipeline}}{\text{Energy available at reservoir}}$$

or

$$\eta_{\text{trans}} = (H_1 - h_f)/H_1 = H/H_1$$

and

$$\text{Nozzle efficiency} = \frac{\text{Energy at nozzle outlet}}{\text{Energy at nozzle inlet}}$$

or

$$\eta_N = H'/(H_1 - h_f) = C_1^2/2gH$$

So

$$\text{Nozzle and pipe transmission efficiency} = (H/H_1)(H'/H) = C_1^2/2gH_1$$

Also

$$\text{Nozzle velocity coefficient} = \frac{\text{Actual jet velocity}}{\text{Theoretical jet velocity}}$$

or

$$C_v = C_1/(2gH)^{1/2}$$

Therefore the nozzle efficiency becomes

$$\eta_N = C_1^2/2gH = C_v^2$$

3-b

The hydraulic efficiency is given by

$$\eta_H = \frac{\text{Power transferred to runner}}{\text{Power available}}$$

$$\text{Power transferred to runner} = \rho g Q H \eta_H$$
$$= W\eta_H$$

But from Euler's turbine equation (Eq. (1.24))

$$E = \frac{W}{mg} = \frac{U_1C_{x1} - U_2C_{x2}}{g}$$

Therefore

$$\eta_H gH = U_1C_{x1} - U_2C_{x2}$$

and $C_{x2} = 0$. Hence

$$C_{x1} = \frac{0.9 \times 9.81 \times 62}{U_1}$$

Now

$$U_1 = \frac{\pi ND}{60}$$
$$= \frac{\pi \times 375 \times 1.5}{60}$$
$$= 29.45 \text{ m/s}$$

Therefore

$$C_{x1} = \frac{0.9 \times 9.81 \times 62}{29.45}$$
$$= 18.58 \text{ m/s}$$

$$P^{1/2} = \frac{0.14 \times (10^3)^{1/2} \times (9.81 \times 62)^{5/4} \times 60}{375}$$

$$= 2140$$

P = 4578 kW (this power is delivered to runner)

Thus

$$0.9 = \frac{4578 \times 10^3}{\rho g Q H}$$

Flow rate

$$Q = \frac{4578 \times 10^3}{10^3 \times 9.81 \times 62}$$

$$= 7.53 \text{ m}^3/\text{s}$$

$$\text{Flow area} = \frac{\text{Flow rate}}{\text{Flow velocity}}$$

$$= \frac{Q}{C_2} \qquad \text{since } C_2 = C_{r2}$$

and at exit from the runner the flow area may be written in terms of the runner exit diameter and runner height b_2:

$$\pi d_2 b_2 = Q/C_2$$

where d_2 is the draft tube entry diameter. Now the runner height at entry b_1 is given by

$$b_1 = \frac{Q}{\pi d_1 C_{r1}}$$

$$= \frac{7.53}{\pi \times 1.5 \times 9}$$

$$= 0.178 \text{ m}$$

Also

$$b_2 = 2 + \frac{b_1}{2} - 1.7$$

$$= 2 + 0.089 - 1.7$$

$$= 0.389 \text{ m}$$

Substituting for b_2 we get

$$d_2 = \frac{7.53}{\pi \times 0.389 \times 7}$$

Draft tube diameter $= 0.88$ m

4-a)

When the flow is choked, $C^2 = a^2 = \gamma RT$. Since $h_0 = h + \frac{1}{2}C^2$, then $C_p T_0 = C_p T + \frac{1}{2}\gamma RT$, and

$$\frac{T}{T_0} = \left(1 + \frac{\gamma R}{2C_p}\right)^{-1} = \frac{2}{\gamma + 1}$$

Assuming isentropic flow, we have:

$$\left(\frac{\rho}{\rho_0}\right) = \left(\frac{P}{P_0}\right)\left(\frac{T_0}{T}\right) = \left[1 + \frac{1}{2}(\gamma - 1)M^2\right]^{(1-\gamma)/(\gamma-1)}$$

and when $C = a$, $M = 1$, so that:

$$\left(\frac{\rho}{\rho_0}\right) = \left[\frac{2}{(\gamma + 1)}\right]^{1/(\gamma-1)}$$

Using the continuity equation, $\left(\frac{\dot{m}}{A}\right) = \rho C = \rho\left[\gamma RT\right]^{1/2}$, we have

$$\left(\frac{\dot{m}}{A}\right) = \rho_0 a_0 \left[\frac{2}{\gamma+1}\right]^{(\gamma+1)/2(\gamma-1)}$$

where (ρ_0 and a_0 refer to inlet stagnation conditions, which remain unchanged. The mass flow rate at choking is constant.

4-b)

The impeller tip speed is given by:

$$U_2 = \frac{\pi DN}{60} = \frac{(\pi)(1)(5945)}{60} = 311 \text{ m/s}$$

The work input: $W = \sigma U_2^2 = \frac{(0.9)(311^2)}{1000} = 87 \text{ kJ/kg}$

Using the isentropic P–T relation and denoting isentropic temperature by $T_{3'}$, we get:

$$T_{3'} = T_1 \left(\frac{P_3}{P_1}\right)^{0.286} = (298)(2.2)^{0.286} = 373.38 \text{ K}$$

Hence the isentropic temperature rise:

$$T_{3'} - T_1 = 373.38 - 298 = 75.38 \text{ K}$$

The temperature equivalent of work done:

$$T_3 - T_1 = \left(\frac{W}{C_p}\right) = 87/1.005 = 86.57 \text{ K}$$

The compressor adiabatic efficiency is given by:

$$\eta_c = \frac{(T_{3'} - T_1)}{(T_3 - T_1)} = \frac{75.38}{86.57} = 0.871 \text{ or } 87.1\%$$

The air temperature at the impeller exit is:

$$T_3 = T_1 + 86.57 = 384.57 \text{ K}$$

Power input:

$$P = \dot{m}W = (28)(87) = 2436 \text{ kW}$$

c)

(a) The axial velocity is first found from the velocity triangles and since they are symmetrical for 50 per cent reaction $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$. Therefore

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1$$

$$C_a = \frac{200}{\tan 45 + \tan 13}$$

$$= 162.5 \text{ m/s}$$

Work done in the first stage is given by

$$C_p(T_{03} - T_{01}) = \psi C_a U(\tan\beta_1 - \tan\beta_2)$$
$$= 0.86 \times 162.5 \times 200(\tan 45 - \tan 13)$$
$$= 21\,497 \text{ J/kg}$$

For the first stage, the pressure ratio is given by

$$\frac{p_{03}}{p_{01}} = \left(1 + \eta_c \frac{T_{03} - T_{01}}{T_{01}}\right)^{\gamma/(\gamma-1)}$$
$$= \left(1 + \frac{0.84 \times 21\,497}{288 \times 1005}\right)^{3.5}$$
$$= 1.24$$

To find the overall static pressure ratio, we may use

$$\frac{p_{II}}{p_I} = \left(\frac{T_{II}}{T_I}\right)^{\eta_{\infty}\gamma/(\gamma-1)}$$

Now

$$T_{0II} = T_{0I} + 10 \times \Delta T_0$$
$$= 288 + \frac{10 \times 21\,497}{1005}$$
$$= 502 \text{ K}$$

But

$$C_1 = \frac{C_a}{\cos\alpha_1}$$
$$= \frac{162.5}{\cos 13}$$
$$= 166.8 \text{ m/s}$$

Therefore

$$T_I = T_{0I} - \frac{C_1^2}{2C_p}$$
$$= 288 - \frac{166.8^2}{2 \times 1005}$$
$$= 274.1 \text{ K}$$

and

$$T_{II} = T_{0II} - \frac{C_1^2}{2C_p}$$
$$= 502 - \frac{166.8^2}{2 \times 1005}$$
$$= 488.1 \text{ K}$$

Thus

$$\frac{p_{II}}{p_I} = \left(\frac{488.1}{274.1}\right)^{0.88 \times 1.4/0.4}$$

Overall static pressure ratio = 5.91

5-a) **Reaction Blading**

The pressure reduces through succeeding stator and rotor rows. The velocity being recovered as the pressure drops, and this necessitates a blade passage that is convergent towards the outlet as shown in Figure 5 . It is noted that the inlet angle β_1 for reaction blade is almost zero while the profile of the back of the blade is almost linear. In reaction turbine blades with reaction ratio greater than 50% the blades should be twisted to attain high efficiency. While in reaction turbine stage with reaction ratio of 50% or less and for zero reaction and impulse turbine, using straight untwisted blades will not affect the efficiency so much. Reaction blading is used at low pressure end steam turbine and in gas turbine

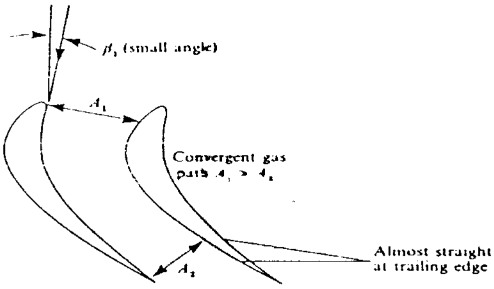


Figure 5: reaction blading

Impulse Blading

Impulse blading is employed successfully at high pressure end of steam turbine. The velocity of the steam is increased in the convergent nozzle to perhaps 800 m/s before entering the rotor blades and passing through them at constant pressure. The rotor area passages are usually of constant-area symmetrical cross-section, with inlet and outlet angles of 45° (β_1 and β_2) being typical Figure 6.

The centers of curvature of the convex and concave surfaces of adjacent blades are then located at the same point to form parallel passages.

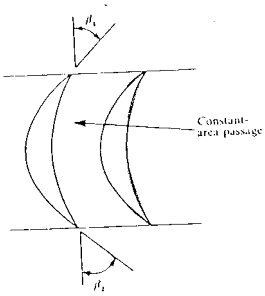


Figure 6: Impulse blading

In the impulse stage, the total pressure drop occurs across the stationary blades (or nozzles). This pressure drop increases the velocity of the steam. However, in the reaction stage, the total pressure drop is divided equally across the stationary blades and the moving blades. The pressure drop again results in a corresponding.

As shown in Figs 7, the shape of the stationary blades or nozzles in both stage designs is very similar. However, a big difference exists in the shapes of the moving blades. In an impulse stage, the shape of the moving blades or buckets is like a cup. The shape of the moving blades in a reaction stage is more like that of an airfoil. These blades look very similar to the stationary blades or nozzles. increase in the velocity of the steam flow.

Most of the steam turbine plants use impulse steam turbines, whereas gas turbine plants seldom do. The general principles are the same whether steam or gas is the working substance.

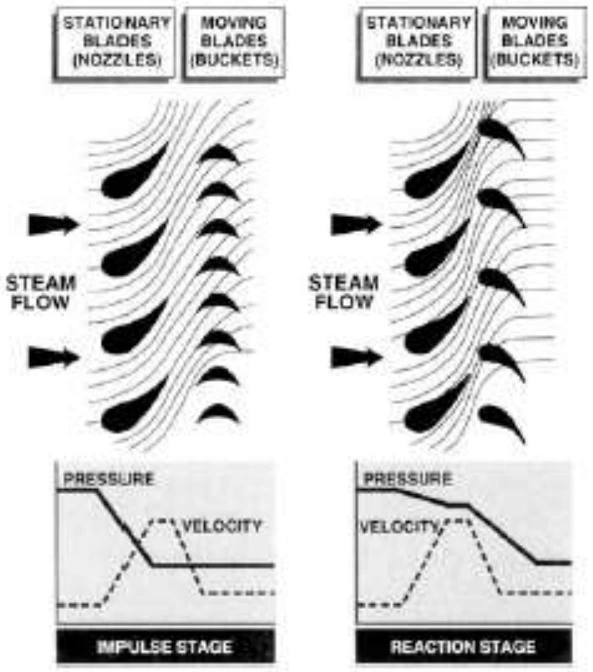
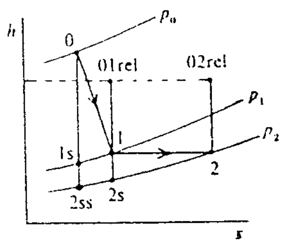
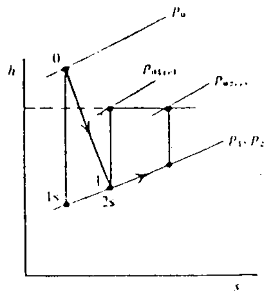


Figure 7 Impulse and reaction stage design.

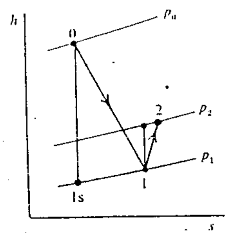
b) Mollier curves for :



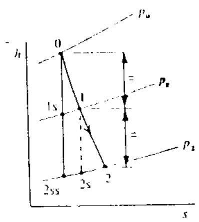
a) Zero reaction axial stage



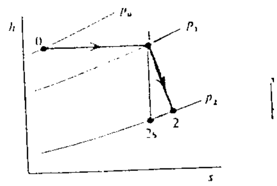
b) Pure impulse stage



c) Negative reaction stage



d) A 50% reaction stage



e) A100% reaction stage

Figure11

5-c) Since the discharge from the wheel is axial

$C_a=C_2$. Then Wx2=U at the mean radius.

$$\text{Torque} = mr(W_{x2} + W_{x1})$$

$$W_{x2} + W_{x1} = \frac{1.62}{0.1325 \times 0.014}$$

$$= 873.3 \text{ m/s}$$

Since R = 0 $\rightarrow \beta_2 = 60.7^\circ$

(b) Also from the velocity diagrams

$$C_1^2 = (W_{x1} + W_{x2})^2 + C_{a1}^2$$

$$= 873.3^2 + 317.9^2$$

$$= 8.637 \times 10^5 \text{ m}^2/\text{s}^2$$

whence

$$W = mU(W_{x2} + W_{x1})$$

$$U = \frac{3.75 \times 10^3}{0.014 \times 873.3}$$

$$= 306.7 \text{ m/s}$$

Now

$$C_{a1} = \frac{W_{x1} + W_{x2}}{\tan 70^\circ}$$

$$= \frac{873.3}{\tan 70^\circ}$$

$$= 317.9 \text{ m/s}$$

$$W_{x1} = 873.29 - 306.7$$

$$= 566.6 \text{ m/s}$$

Therefore

$$\tan \beta_1 = W_{x1}/C_{a1}$$

$$= \frac{566.6}{317.9}$$

$$\beta_1 = 60.7^\circ$$

$$\text{Diagram efficiency} = \frac{U(W_{x2} + W_{x1})}{C_1^2/2}$$

$$= \frac{306.7 \times 873.3 \times 2}{863\,700}$$

$$= 0.62$$

(c) If there is an axial thrust $C_{a1} \neq C_{a2}$

$$C_{a2} = \frac{W_{x2}}{\tan \beta_2}$$

$$= \frac{U}{\tan \beta_2} \quad (U = W_{x2} \text{ since } C_2 \text{ is axial})$$

$$= \frac{306.7}{\tan 60.7^\circ}$$

$$= 172.1 \text{ m/s}$$

Axial thrust $= m(C_{a1} - C_{a2})$ (Eq. (1.23) in axial direction)

$$= 0.014(317.9 - 172.1)$$

$$= \underline{2.04 \text{ N}}$$

(d) Tangential thrust on blades $= m(W_{x2} + W_{x1})$ (Eq. (1.23))

$$= 0.014 \times 873.3$$

$$= \underline{12.23 \text{ N}}$$