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Benha University, Faculty of Engineering	**Elect.Eng.Depart. Third Year 2013**
Subject: Control Engineering (E352)	**Time: 3-hours**

Answer

Q1 a- Define: ω_n , ω_d , ω_r , ω_B , ω_c , ω_g , ω_p , M_r, η, G_m, γ_m? (15 marks)

-Natural frequency ω_n rad/sec**:** it is the natural frequency depends on the natural of the system parameters.

- Under damped natural frequency ω_d rad/sec**:** it is the under damped natural frequency depends on the damping coefficient η as it is less than one $\eta < 1$.

-Resonant frequency ω_r rad/sec**:** it is the frequency at which the peak value of the output frequency response for a second order is equal to $\boldsymbol{\omega_r = \omega_n\sqrt{1-2\zeta^2}}$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}, \quad \text{for } 0 \le \zeta \le 0.707$$

As ζ approaches zero, M_r approaches infinity

$0 < \zeta \le 0.707$, the resonant frequency ω_r is less than the damped natural frequency

-Cut off frequency ω_B rad/sec**:** it is the frequency at which the magnitude of the output frequency response is equal to $(=\frac{1}{\sqrt{2}})\ of\ the\ low\ frequency$.

-Corner frequency ω_c rad/sec**:** it is the frequency at which the magnitude of the output frequency response is changed sharply. It may be (0, 1, 1/T, ω_n)

-Gain crossover frequency ω_g: it is the frequency at which the magnitude of the output frequency response is equal to one or zero decibel.

$|G(j\,\boldsymbol{\omega g})H(j\,\boldsymbol{\omega g})| = 1$ or $|G(j\,\boldsymbol{\omega g})H(j\,\boldsymbol{\omega g})| = 0db$

-Phase crossover frequency ω_p: it is the frequency at which the phase of the output frequency response is equal to (-180) degrees.

Imag. $[G(j\,\omega_p)H(j\,\omega_p)]=0$ or $\angle G(j\,\boldsymbol{\omega p})H(j\,\boldsymbol{\omega p}) = -180\text{deg}.$

-Maximum resonant magnitude M_r: it is the peak value of the output frequency response for a second order system $\boldsymbol{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad\qquad M_r = |G(j\omega)|_{max} = |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

-damping coefficient η it depends on the natural of the system parameters. For second order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Values of η	**System stability**	**Step-response**
$0 > \eta$	System is unstable	undefined
$\eta = 0$	System is critically stable	oscillatory
$0 < \eta < 1$	System is stable	Under-damped
$0 < \eta = 1$	System is stable	Critically damped
$0 < \eta > 1$	System is stable	Over damped

-Gain margin G_m: it is reciprocal of the magnitude of the output frequency response at the **Phase crossover frequency ω_p**

$\mathbf{G_m}$=1/[Real of G(j ω_p)H(j ω_p)]= 1/|G(j $\boldsymbol{\omega p}$)H(j $\boldsymbol{\omega p}$)| =K_c/K

G_M=20log G_m db

-Phase margin γ_m: it is the angle of the output frequency response at the **gain crossover** frequency plus 180 degrees.

$$\gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180 \text{ deg.}$$

b- Consider a control system shown in Fig.1 if **G(S) =9/[S(S+3)], H(S) =1**

i-Find the **frequency response** as r(t)=5sinωt **?** ii-Calculate **M_r, ω_r?**

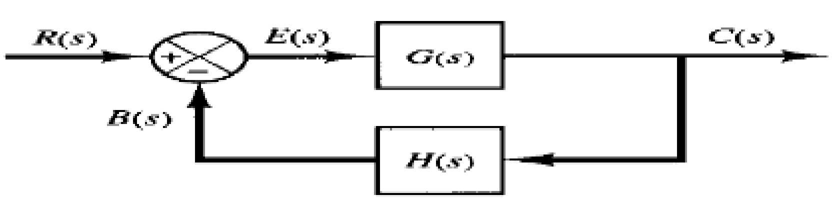


Fig.1

Frequency Response: it means the steady state output of a 1-linear 2-time-invariant 3- stable control system to a sinusoidal input and it is a sinusoidal with phase shift positive or negative and does not depend on the initial conditions.

b-Steps to find frequency Response:

1-the closed loop transfer function =**T(s)=C(S)/R(S)** =

C(S) / R(S) =$\frac{G(s)}{1+G(S)H(S)} = \frac{\omega_n{}^2}{S^2+2\eta\omega_n S+\omega_n{}^2} = \frac{9}{S^2+3S+9}$, $\omega_n = \frac{3\text{rad}}{\text{sec}}$ $\zeta = 0.5$

2-the closed loop frequency transfer function =

T (jω)=C(jω)/R(jω) $= \frac{9}{(j\omega)^2+3(j\omega)+9}$ = M∟Φ=Re+j imag

$$\boldsymbol{M} = \frac{\mathbf{9}}{\sqrt{(\mathbf{9}-\boldsymbol{\omega}^2)^2+\mathbf{9}\boldsymbol{\omega}^2}} \quad , \ \boldsymbol{\Phi} = -\tan^{-1}[3\omega/(9-\omega^2)]$$

3-As the input =$r(t) = 2sin\omega t$ then

the response = $C(t) = 2Msin(\omega t + \Phi)$

$$= \frac{\mathbf{18}}{\sqrt{(\mathbf{9}-\boldsymbol{\omega}^2)^2+\mathbf{9}\boldsymbol{\omega}^2}} \sin[\omega t - \tan^{-1}[3\omega/(9-\omega^2)]$$

$$\boldsymbol{M_r} = \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{\zeta}\sqrt{\mathbf{1}-\boldsymbol{\zeta}^2}} = \frac{\mathbf{1}}{\mathbf{2(0.5)}\sqrt{\mathbf{1}-(\mathbf{0.5})^2}} = 1.155$$

$$\boldsymbol{\omega_r} = \boldsymbol{\omega_n}\sqrt{\mathbf{1}-\mathbf{2}\boldsymbol{\zeta}^2} = \mathbf{3}\sqrt{\mathbf{1}-\mathbf{2(0.5)}^2} = \mathbf{2.12}\ \boldsymbol{rad/sec}.$$

Q2 (15 marks)

Consider a control system shown in Fig.1 if **G(S) =K/[(S+3)(S+2)], H(S) =1/S**

a- Sketch the **complete root locus** for positive values of **K**?

b- Find **K** that makes the complex closed loop poles have a damping ratio =**0.6** and **find the closed loop poles** using **the plot**?

c- Find **K** that makes the complex closed loop poles have a damping ratio =**0.6** and **find the closed loop poles analytically**?

d- Write short MATLAB program to solve **a** and solve **b**?

Root locus:

1-the root locus is symmetrical about the real axis in the S-plane

2-the open loop TF=G(s) H(s)= **G(S) H(S) =K/[S(S+3)(S+2)]**=K/[S^3+5S^2+6S+0]

3-the root locus starts at the pole and ends at the zero or infinity

4-number of root loci= n=number of poles of the open loop TF =3 at [0,-2,-3]

5-number of zeros= m=0

6-number of asymptotes = n-m=3-0=3

8-center of gravity =point of intersection of asymptotes with real axis=

$$A = \frac{\sum poles - \sum zoles}{n-m} = \frac{-0-2-3}{3} = -1.7$$

9-angles of asymptotes are = $\Theta = \frac{\pm 180(2R+1)}{n-m} = \pm 60, \pm 180$

10- Points of crossing the imaginary axis as Routh test

Charct.equa=1+G(S)H(S)=0= S^3+5S^2+6s+K

S^3	1	6	K≥0, [30-K]/ 5≥0 then 0≤K≤30,Kc=30 $5S^2+30=0$, S=j ω =j$\sqrt{6}$ rad/sec
S^2	5	K	
S	[30-K]/ 5		
S^0	K		

11- break points (break away or break in) at

$$-\frac{dK}{dS} = 0 = \frac{d}{dS}\left[\frac{1}{G(S)H(S)}\right] = \frac{d}{dS}[S^3 + 5S^2 + 6s + 0] = 3S^2 + 10s + 6 = 0$$

S=-2.6 refused, S=-0.8 is a break- away point

12-break angles at [±180(2R+1)/r] where r=number of branches(poles for break away or zeros for break in) R=0,1,----- break angles at [±180]/2=±90

13-there is no angle of departure (complex poles)

14- there is no angle of arrival (complex zeros)

15-sketch the root loci as

16- the damping factor or coefficient ζ is straight line with slope $\Theta = \cos^{-1}\boldsymbol{\zeta}$

with respect to the negative real axis in the S-plane. $\Theta = \cos^{-1} 0.6$ =53.13deg. at the test point (intersection point) S_d=-0.7±j0.8

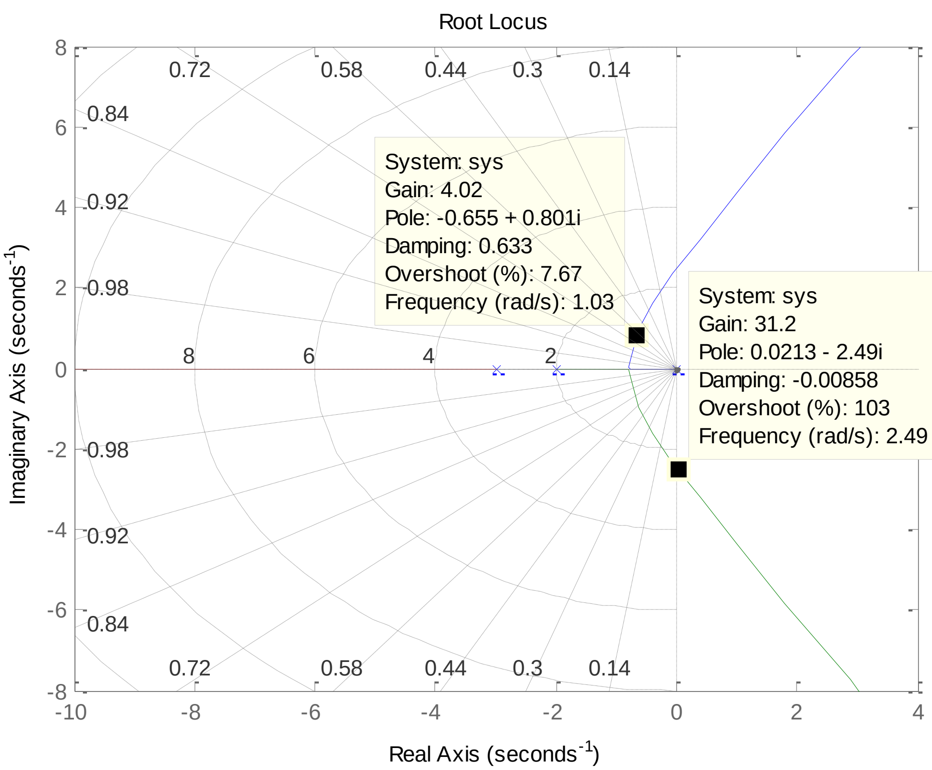
$$\text{angle condition} = \sum_{n=1}^{n=3} [\Theta_{zeros} - \Theta_{poles}] = \pm 180(2R+1) = 128 + 22 + 32 = 182 \text{ deg}$$

$$\text{magnitude condition} = \sum_{n=1}^{n=3} \frac{\|\text{poles}\|}{\|\text{zeros}\|} = K = 2.5 * 1.3 * 1.4 = 4.4$$

$$\sum_{n=1}^{n=3} open\ loop\ poles = \sum_{n=1}^{n=3} closed\ loop\ poles = \text{constant as n} - \text{m} \geq 2$$

$$\sum_{n=1}^{n=3} open\ loop\ poles = -0 - 2 - 3 = -5 = \sum_{n=1}^{n=3} closed\ loop\ poles = 2(-0.7) \pm \text{j}0.8 + \text{p}$$

$$\text{then p} = -3.6 \text{ i.e. closed loop poles are } [-0.7 \pm \text{j}0.8, -3.6\,]$$



19- To find analytically closed loop poles and K as

$(S^2+2\zeta\omega_n S+\omega_n^2)(S+a)$=characteristic equa. for a third order syst.

Solve $1+G(S)H(S)=0= S^3+5S^2+6s+K=(S^2+1.2\omega_n S+\omega_n^2)(S+a)$

$= S^3+(1.2\omega_n+a)S^2+(1.2\omega_n a+\omega_n^2)S+\omega_n^2 a$

$1.2\omega_n+a=5$,,,, $1.2\omega_n a+\omega_n^2=6$,,,, $\omega_n^2 a=k$

Prog. >>n=[1];d=[1 5 6 0]; rlocus(n,d), grid

Q3 (30marks)

Consider a control system shown in Fig.1 if **H(s)=1/S, G(s)=k/(S+2)(s+3)**,

a- Prove that the gain margin=3.52db at 2.45rad/sec. and the phase margin=11.9 degrees at 1.98rad/sec. **as K=20**?
b- Sketch the **polar plot as K=20**?
c- Sketch the **Bode plot as K=20**?
d- Find the gain margin and the phase margin using **the plots**?
e- Write a short MATLAB program to solve b ,c and d?
f- Write short MATLAB program to Sketch the **Nichols plot**?

1- the open loop TF=G(s) H(s)= **G(S) H(S) =K/[S(S+3)(S+2)]**=$20/[S^3+5S^2+6s+20]$
2- Find the freq.open loop TF=

$$\mathbf{G(j\omega)H(j\omega)}=\frac{20}{[(j\omega+3)(j\omega+2)(j\omega)]}=Me^{j\Phi}=M\llcorner\Phi=\text{Re}+j\text{ imag}$$

$$\boldsymbol{M}=\frac{\mathbf{20}}{\boldsymbol{\omega}\sqrt{\mathbf{4+\omega^2}}\,\sqrt{\mathbf{9+\omega^2}}}\quad,\ \boldsymbol{\Phi}=\mathbf{-90-\tan^{-1}(\omega/2)-\tan^{-1}(\omega/3)}]$$

$$M=\frac{20}{\omega\sqrt{4+\omega^2}\,\sqrt{9+\omega^2}}=\frac{20}{1.98\sqrt{4+1.98^2}\,\sqrt{9+1.98^2}}=1$$

$$\boldsymbol{M}=\frac{\mathbf{20}}{\boldsymbol{\omega}\sqrt{\mathbf{4+\omega^2}}\,\sqrt{\mathbf{9+\omega^2}}}=\frac{\mathbf{20}}{1.98\sqrt{\mathbf{4}+1.98^2}\,\sqrt{\mathbf{9}+1.98^2}}=\mathbf{1}$$

$$\boldsymbol{M}=\frac{\mathbf{20}}{\boldsymbol{\omega}\sqrt{\mathbf{4+\omega^2}}\,\sqrt{\mathbf{9+\omega^2}}}=\frac{\mathbf{20}}{\mathbf{2.45\sqrt{4+2.45^2}\,\sqrt{9+2.45^2}}}=\mathbf{0.67}$$

M=20log(1/0.67)=3.52db

$$\boldsymbol{\Phi}=\mathbf{-90-\tan^{-1}(2.45/2)-\tan^{-1}(2.45/3)}]=-180\text{ deg.}$$

$\boldsymbol{\Phi} = -\mathbf{90} - \mathbf{tan}^{-1}(1.98\,/\mathbf{2}) - \mathbf{tan}^{-1}(1.98\,/\mathbf{3})] = -168.12\ \mathbf{deg.}$

$\gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180\text{ deg.} = 180 - 168.12 = 11.88deg.$

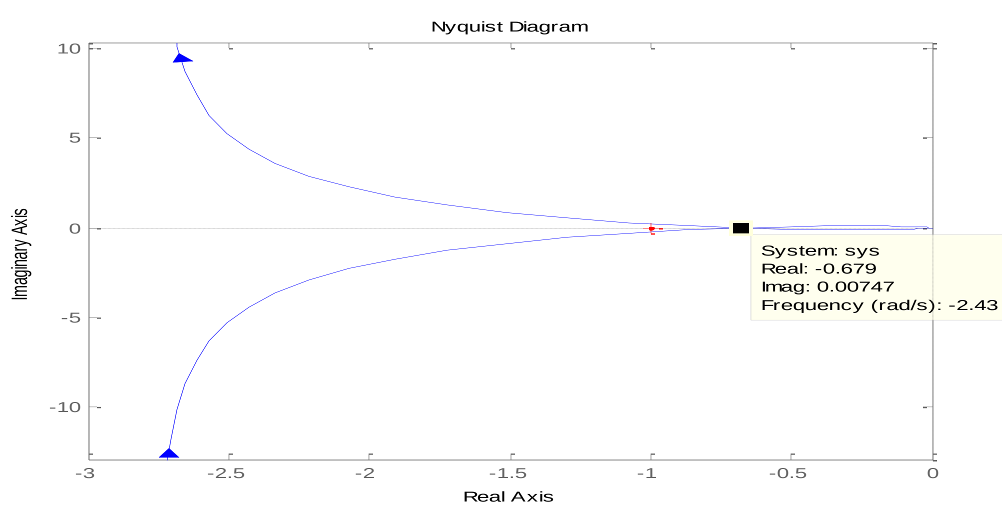
3- Find the table

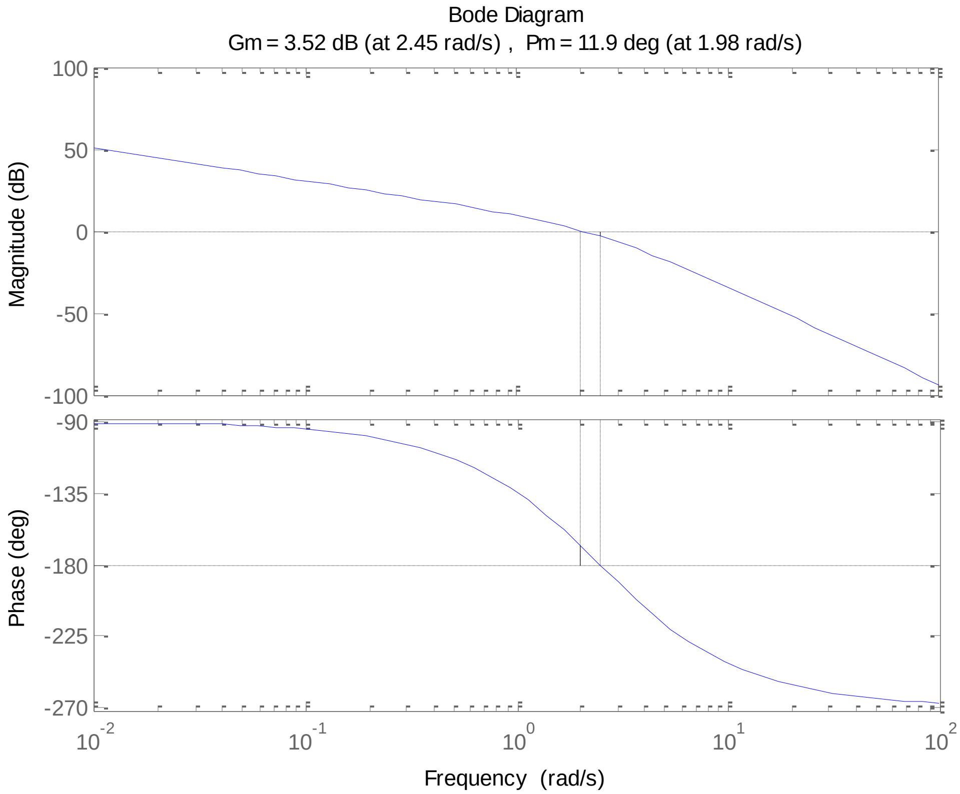
ω	0	0.1	1	2.35	3.32	5	10	∞
Φ	0	-10.5	-90	-155	-180	-206	-236	-270
M	5	4.97	3	1	0.5	0.2	0.03	0
20logM	14	13.9	9.5	0	6.4	-14.6	-31.1	0
Real **G(jω)H(jω)**	5		0					0
Imag **G(jω)H(jω)**	0		-3		0			0

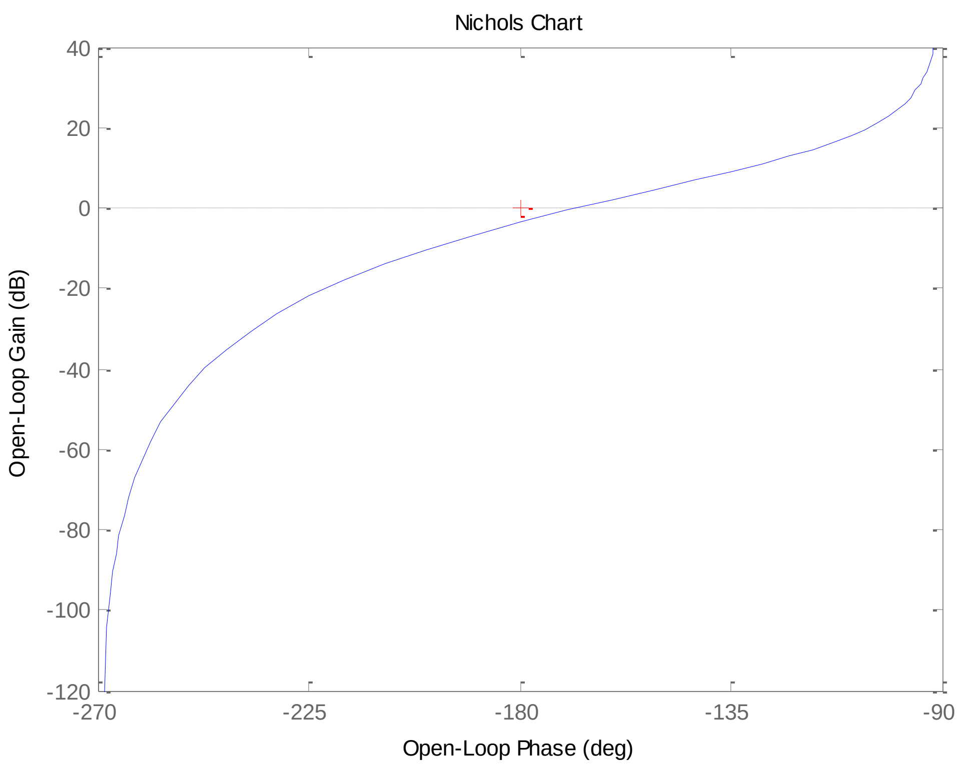
4- Plot the vector on the **jω − plane** where Φ in degrees as a straight line and determine M on this line
5- Plot the locus of the vector as points from the table
6- Find the gain and the phase margins from the plot

Prog. >>n=[20]; d=[1 5 6 0];

>> nyquist(n,d) >> margin(n,d) >> nichols(n,d)




Bode Diagram
Gm = 3.52 dB (at 2.45 rad/s) , Pm = 11.9 deg (at 1.98 rad/s)
Magnitude (dB)
100
50
0
-50
-100
Phase (deg)
-90
-135
-180
-225
-270
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/s)


Nichols Chart
Open-Loop Gain (dB)
40
20
0
-20
-40
-60
-80
-100
-120
-270
-225
-180
-135
-90
Open-Loop Phase (deg)