**Benha University**
**College of Engineering at Benha**
**Mechanical Eng. Dept.**
**Subject :Automatic Control**

**4th Year Mechanics**
**Date 11/5/2013**

# Model Answer of the Final Exam

**Elaborated by: Dr. Mohamed Elsharnoby**

**المادة : التحكم الآلى م 482**

أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

1-a) Find the signal flow graph of the system shown in Figure 1 Use the given variables as nodes. List all loops.

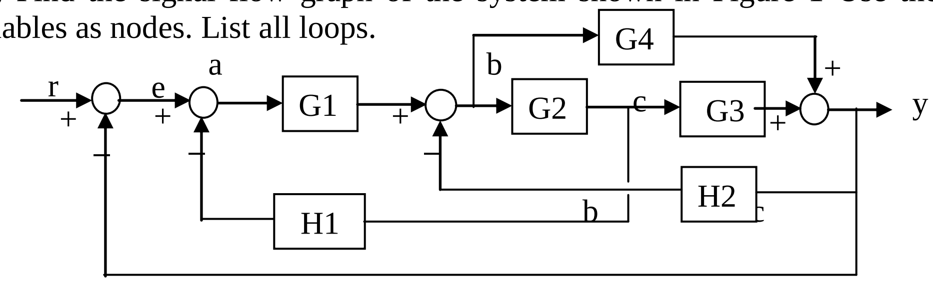
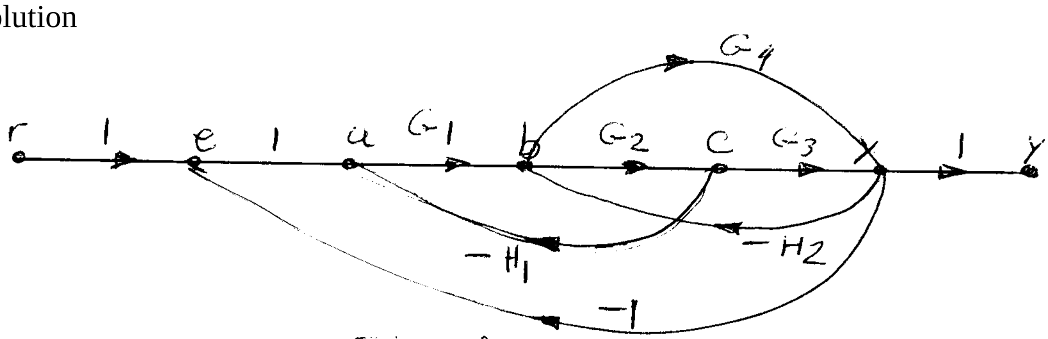



Figure 1

Solution




Signal Flow Graph

Loops

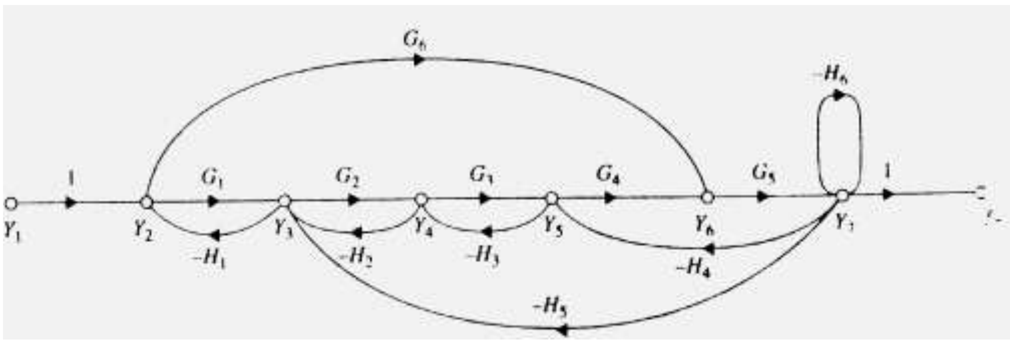
$L_1 = -G_1 G_2 H_1$ , $L_2 = -G_2 G_3 H_2$

$L_3 = -G_4 H_2$ $\quad$ $L_4 = -G_1 G_2 G_3$

$L_5 = -G_1 G_4$

1-b)

The SFG is given by:



Solution

Loops

$L_1 = -G_1 H_1 \quad , \quad L_2 = -G_2 H_2 \quad , \quad L_3 = -G_3 H_3$

$L_4 = -G_4 G_5 H_4 \quad L_5 = -H_6 \quad . \quad L_6 = -G_2 G_3 G_4 G_5 H_5$

Paths

$P_1 = G_1 G_2 G_3 G_4 G_5 \quad , \quad P_2 = G_6 G_5$

$$\Delta = 1 - \sum_{i=1}^{6} L_i + \underset{\text{non touch}}{\sum L_i L_j} - \underset{\text{non touching}}{\sum L_i L_j L_k}$$

$$= 1 + G_1 H_1 + G_2 H_2 + G_3 H_3 + G_4 G_5 H_4 + H_6 + G_2 G_3 G_4 G_5 H_5$$

$$+ L_1 L_3 + L_1 L_4 + L_1 L_5 + L_2 L_4 + L_2 L_5 + L_3 L_5 + L_3 L_6$$

$$- L_1 L_3 L_5$$

$$\Delta = 1 + (G_1 H_1 + G_2 H_2 + G_3 H_3 + G_4 G_5 H_4 + H_6 + G_2 G_3 G_4 G_5 H_5)$$

$$+ G_1 H_1 G_3 H_3 + G_1 H_1 G_4 G_5 H_4 + G_1 H_1 H_6 + G_2 H_2 G_4 G_5 H_4$$

$$+ G_2 H_2 H_6 + G_3 H_3 H_6 + G_2 G_3^2 G_4 G_5 H_3 H_5$$

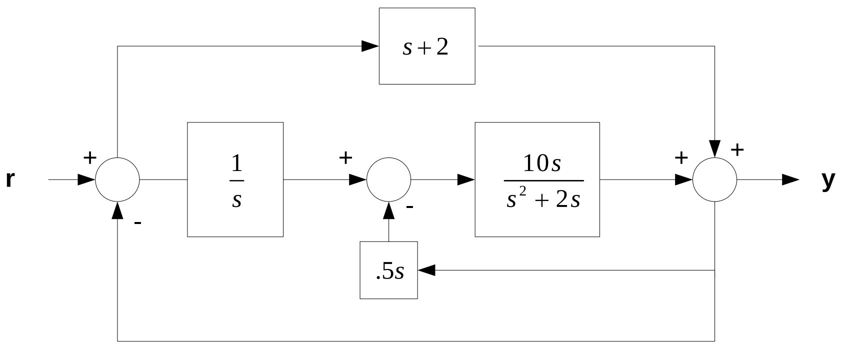
$$+ G_1 H_1 G_3 H_3 H_6$$

$\Delta_1 = 1 \qquad \Delta_2 = 1 + G_2 H_2 + G_3 H_3$

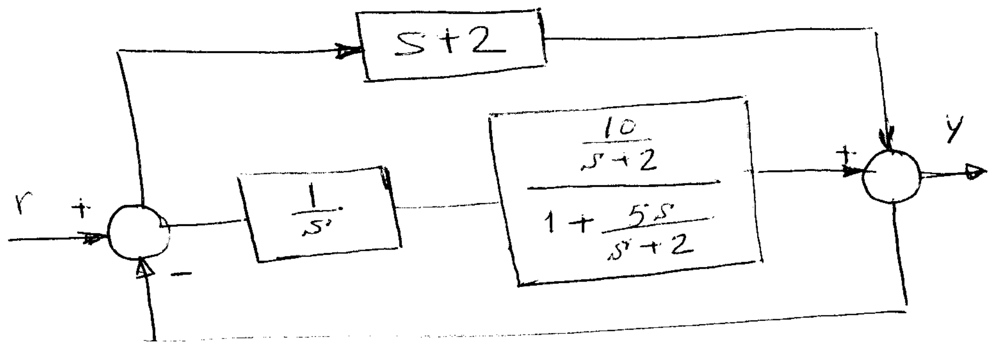
$$TF = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 G_4 G_5 + G_5 G_6 (\Delta_2)}{\Delta}$$

2-a)

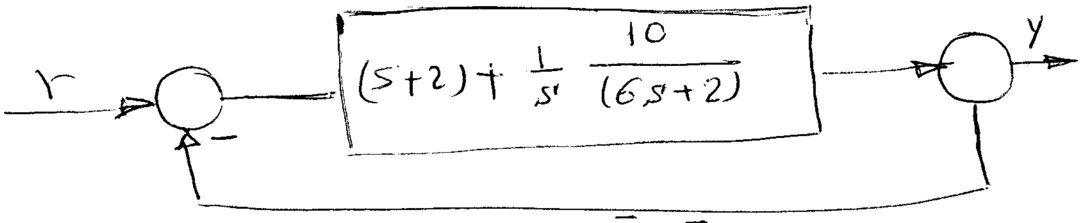
We want the transfer function of



The Block $\frac{10s}{s^2+2s} = \frac{10}{s+2}$




The system is reduced to




(i) $$T.F = \frac{(s+2) + \frac{5}{3s^2+s}}{1+(s+2)+\frac{5}{3s^2+s}}$$

$$= \frac{3s^3+7s^2+2s+5}{3s^3+10s^2+3s+5}$$

If r is unit step, The open loop T.F

$$G(s) = (s+2) + \frac{1}{s}\left(\frac{10}{6s+2}\right)$$

(ii) $K_p = \lim_{s\to 0} G(s) = \infty \Rightarrow e_{ss} = \frac{1}{1+K_p} = 0$

(iii) $K_v = \lim_{s\to 0} sG(s) = 5 \Rightarrow e_{ss} = \frac{1}{5} = 20\%$

Construct The Howartz array

| | | |
|---|---|---|
| $s^3$ | 3 | 3 |
| $s^2$ | 10 | 5 |
| $s$ | 1.5 | 0 |
| 1 | 5 | 0 |

The system is BIBO stable.

2-b) Hand sketch the root locus of $1 + KG(s) = 0$ as K varies from 0 to $+\infty$, where

$$G(s) = \frac{s+2}{s(s+1)(s+3)^2}$$

open loop poles are 0, -1, -3, -3

" " zero are -2 =

Number of asymptotes are $n - m = 3$

asymptotes angle $= \frac{(2k+1)\pi}{3} = 60^\circ, 180^\circ, 300^\circ$

Intersection of Asymptotes on the real line

$$\alpha = \frac{\sum \text{poles} - \sum \text{zeros}}{n-m} = \frac{-1, -3 -3 + 2}{3} = \frac{-5}{3}$$

The characteristic eqn. is

$$1 + KG(s) = 0 \Rightarrow 1 + \frac{K(s+2)}{(s^2+s)(s^2+6s+9)} = 0$$

$$s^4 + 7s^3 + 15s^2 + 9s + Ks + 2K = 0$$

Construct the array.

| | | | | |
|---|---|---|---|---|
| $s^4$ | $1$ | $15$ | $2K$ | |
| $s^3$ | $7$ | $(9+K)$ | $0$ | |
| $s^2$ | $\frac{105-9K-9}{7}$ | | $2K$ | |
| $s$ | $\frac{(96-9K)(9+K) - 98K}{96-9K}$ | | | $0$ |
| $1$ | $2K$ | | | |

$$96 - 9K > 0 \Rightarrow K < \frac{96}{9}$$

$$(96 - 9K)(9+K) - 98K > 0 \Rightarrow$$

$$864 - 81K - 98K - 9K^2 > 0$$

$$864 - 179K - 9K^2 > 0 \quad K \leq 4,$$

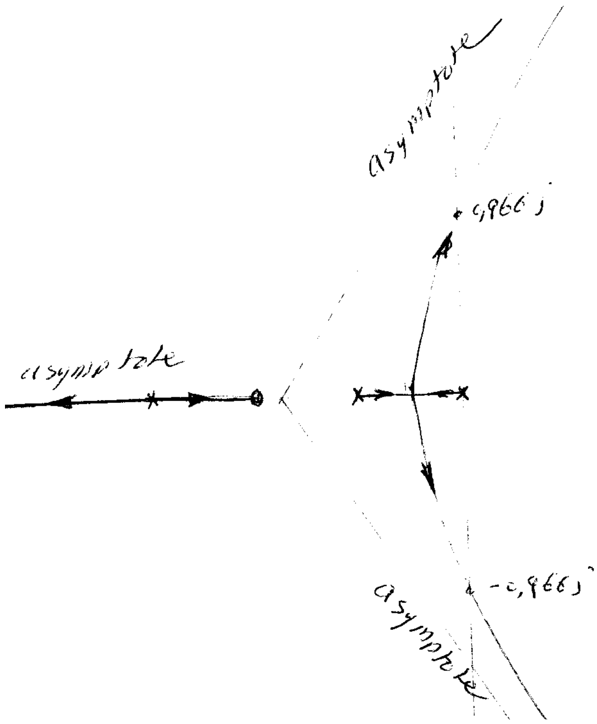
For $K = 4$

The Auxiliary Eqn. is $\frac{60}{7}s^2 + 8 = 0$

$$s = \pm\sqrt{\frac{14}{15}}\, j$$

$$= \pm 0.966 j$$

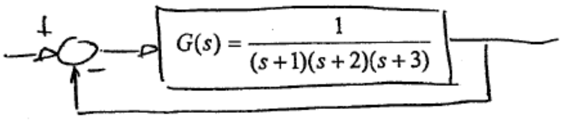
points intersect the Imaginary line.




The root locus

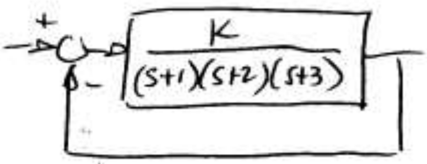
3-a)

Given the control system




**(i)** Analytically determine, as shown in class, the 180 degree phase crossover point of the Nyquist diagram and the corresponding frequency at the 180 degree phase crossover point.

**(ii)** Assuming that the 180 degree phase crossover point is –0.5, what would the gain margin of the system be?




$1 + \frac{K}{(s+1)(s+2)(s+3)} = 0$

$\Rightarrow 1s^3 + 6s^2 + 11s + (6+K)$

(a)

| | | |
|---|---|---|
| $s^3$ | 1 | 11 |
| $s^2$ | 6 | $6+K$ |
| $s^1$ | $60-K$ ← | |
| $s^0$ | $6+K$ | |

$60 - K = 0 \Rightarrow K = 60$

Aux. Eq.: $6s^2 + 6 + 60 = 0$

$s^2 + 11 = 0 \Rightarrow$

$s = \pm j\sqrt{11}$

$-\frac{1}{60}$ = THE 180° PHASE CROSSOVER POINT. $= \underline{-0.0167}$

($\omega = \sqrt{11}$
$= 3.31$)

(b) IF WE HAVE


$(G.M.)(-0.5) = -1 \Rightarrow \underline{Gm = 2}$

3-b)
4-a)

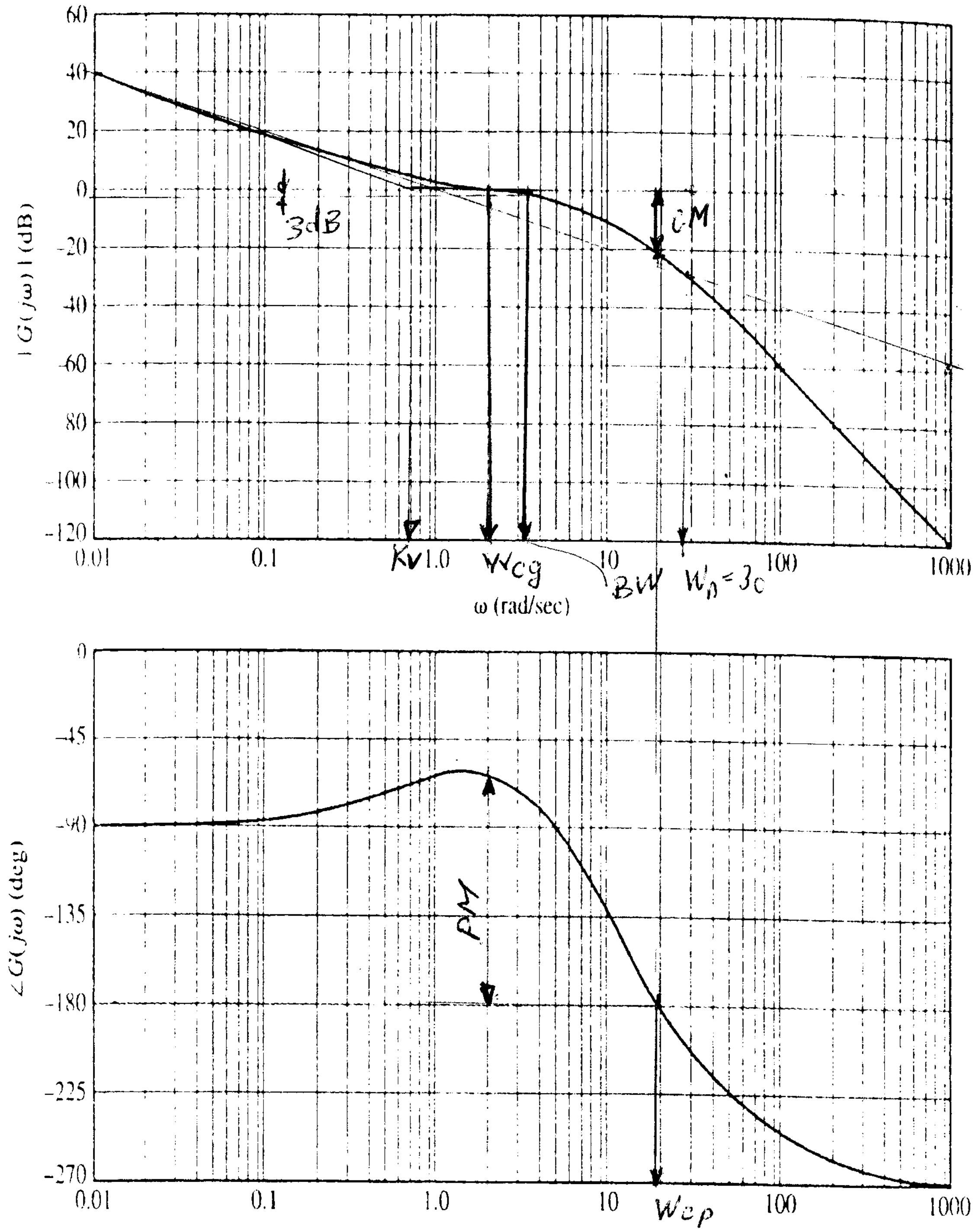



Figure 4

$W_{cg} = 2$ rad/sec

$W_{cp} = 19.5$ rad/sec

$GM = 22$ dB

$PM = 115°$

$w_n = 30$ rad/sec

$Kv = 0.9 \sim 1.0$

system type 1

$e_{ss} = \frac{1}{Kv} \approx 1 \sim 1.11$

$BW = 2.4$ rad/sec.

$BW = 2.4$ rad/sec

ii) The closed loop system as stable when applying Niquest Criteria
As GM is positive, the polar plot does not contain the critical point.
All required values are above

4-a) 3-b) For the characteristic equation of feed back system given by:

$$s^4 + Ks^3 + 5s^2 + 10s + 10K = 0$$

Determine the range of k so that the system is stable.

Construct Routh array.

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 5 | 10K |
| $s^3$ | K | 10 | 0 |
| $s^2$ | $\frac{5K-10}{K}$ | 10K | |
| $s^1$ | $10-\frac{10K^3}{5K-10}$ | 0 | |
| 1 | 10K | | |

① For stable system

$5K-10>0 \quad K>2$

② $10-\frac{2K^3}{K-2}>0$

Condition 2 will not be statified for any K $\longrightarrow$ The system is unstable for any K

4-b) For the system given below in figure 5, estimate the values of K and $K_t$ so that a maximum percentage overshoot of 9.6% and a settling time of 0.05 sec for a tolerance band of 1% are achieved.

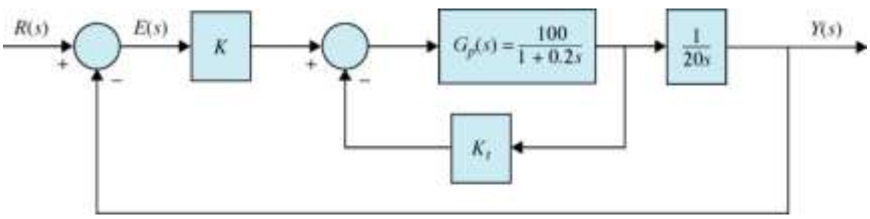
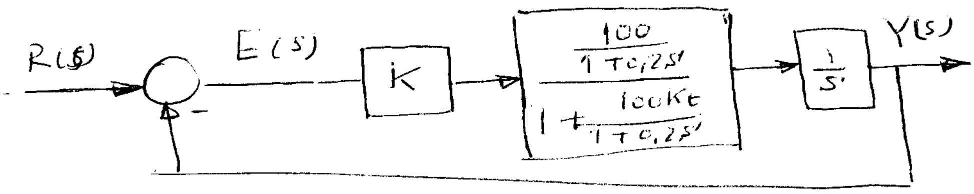
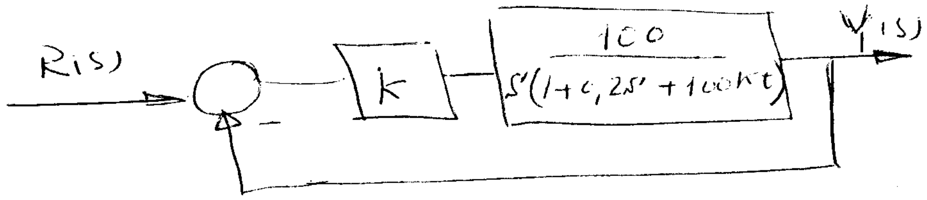
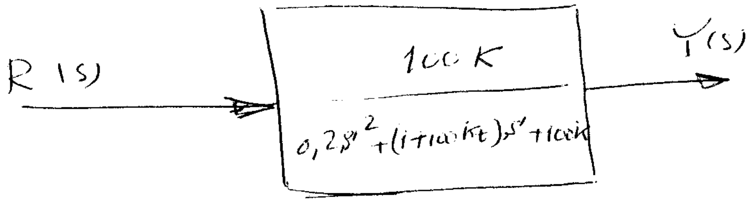


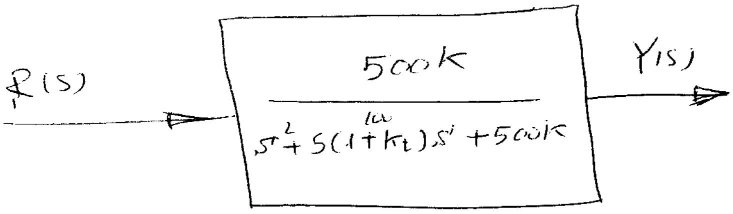

Figure 5

The system is reduced to






$$\omega_n^2 = 500k$$

$$2\xi\omega_n = (5 + 500k_t)$$

for 1% settling time we know

$$t_s = \frac{4.6}{\xi\omega_n} = 0,05$$

$$\therefore \xi\omega_n = 92$$

$$\boxed{\therefore K_t = 0,358}$$

$$M_p = \frac{1}{2\xi\sqrt{1-\xi^2}} = 0.096$$

$$= e^{-\pi\xi/\sqrt{1-\xi^2}} = 0,096$$

$$\xi = 0,5979 \Longrightarrow \omega_n = 153.87$$

$$\boxed{k = \frac{\omega_n^2}{500} = 47.35}$$

5) Consider the standard feedback system shown below in figure 6.

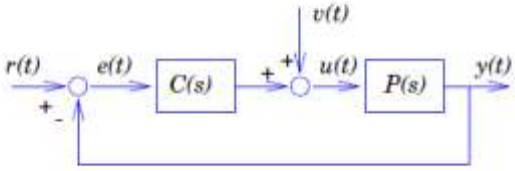
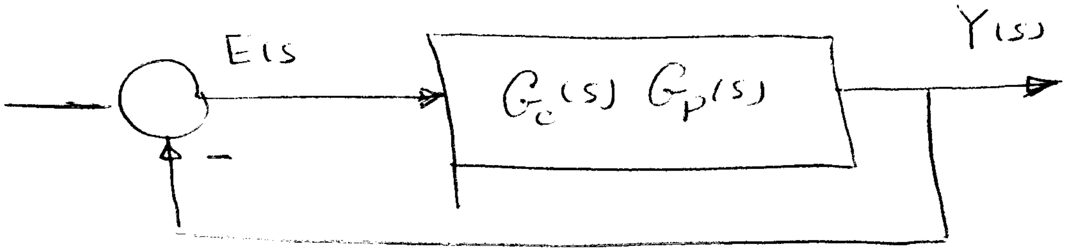



Figure 6




The characteristic Eqn. is given by:

$$1 + G_c(s)\, G_p(s) = 0$$

$$1 + \frac{K_p s + K_i}{s} \cdot \frac{1}{(s^2 + 5s)} = 0$$

$$s^3 + 5s^2 + K_p s + K_i = 0$$

$s = -4$ satisfies the eqn.

$$-64 + 80 - 4K_p + K_i = 0$$

$$16 - 4K_p + K_i = 0$$

$$K_i = 4K_p - 16$$

$$s^3 + 5s^2 + K_p s + 4K_p - 16 = 0$$

$$K_p > 4$$

$$(s^3+4s^2)+(s^2-16)+k_p(s+4)=0$$

$$s^2(s+4)+(s+4)(s-4)+k_p(s+4)=0$$

$$(s+4)\left[s^2+s+(k_p-4)\right]=0$$

$$r_{2,3}=\frac{-1\pm\sqrt{1+16-4k_p}}{2}$$

$$-\frac{1}{2}\pm j\sqrt{k_p-\frac{17}{4}}=-\xi\omega_n\pm j\,\omega_n\sqrt{1-\xi^2}$$

choose $\omega_n=2 \longrightarrow \xi=0{,}25$

$$4(1-\xi^2)=k_p-\frac{17}{4}\Rightarrow \frac{15}{4}+\frac{17}{4}=k_p\Rightarrow$$

$$\boxed{k_p=8}$$

The transfer function $\dfrac{1}{s(s+5)}$

$$=\frac{1}{5}\,\frac{1}{(s)}\,\frac{1}{(0{,}2s+1)}$$