Model Answer Radio Waves course E9413

Q1

I:

a) α

$$\alpha = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left[\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2}-1\right]^{1/2}$$

$$= (64\times 10^6)\sqrt{\frac{(2.25\times 10^{-6})(9\times 10^{-12})}{2}}\left[\sqrt{1+(.867)^2}-1\right]^{1/2} = \underline{0.116\ \text{Np/m}}$$

b) β:

$$\beta = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left[\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2}+1\right]^{1/2} = \underline{.311\ \text{rad/m}}$$

c) $v_p = \omega/\beta = (64\times 10^6)/(.311) = \underline{2.06\times 10^8\ \text{m/s}}$.

d) $\lambda = 2\pi/\beta = 2\pi/(.311) = \underline{20.2\ \text{m}}$.

e) η:

$$\eta = \sqrt{\frac{\mu}{\epsilon'}}\frac{1}{\sqrt{1-j(\epsilon''/\epsilon')}} = \sqrt{\frac{2.25\times 10^{-6}}{9\times 10^{-12}}}\frac{1}{\sqrt{1-j(.867)}} = 407+j152 = \underline{434.5e^{j.36}\ \Omega}$$

f) $\mathbf{H}_s$: With $\mathbf{E}_s$ in the positive y direction (at a given time) and propagating in the positive x direction, we would have a positive z component of $\mathbf{H}_s$, at the same time. We write (with $jk = \alpha + j\beta$):

$$\mathbf{H}_s = \frac{E_s}{\eta}\mathbf{a}_z = \frac{300}{434.5e^{j.36}}e^{-jkx}\mathbf{a}_z = 0.69e^{-\alpha x}e^{-j\beta x}e^{-j.36}\mathbf{a}_z$$
$$= \underline{0.69e^{-.116x}e^{-j.311x}e^{-j.36}\mathbf{a}_z\ \text{A/m}}$$

g) $\mathbf{E}(3,2,4,10\text{ns})$: The real instantaneous form of $\mathbf{E}$ will be

$$\mathbf{E}(x,y,z,t) = \text{Re}\left\{\mathbf{E}_s e^{j\omega t}\right\} = 300e^{-\alpha x}\cos(\omega t - \beta x)\mathbf{a}_y$$

Therefore

$$\mathbf{E}(3,2,4,10\text{ns}) = 300e^{-.116(3)}\cos[(64\times 10^6)(10^{-8}) - .311(3)]\mathbf{a}_y = \underline{203\ \text{V/m}}$$

II:

The electric field of a uniform plane wave in free space is given by $\mathbf{E}_s = 10(\mathbf{a}_y + j\mathbf{a}_z)e^{-j25x}$.

a) Determine the frequency, f: Use

$$f = \frac{\beta c}{2\pi} = \frac{(25)(3 \times 10^8)}{2\pi} = \underline{1.2 \text{ GHz}}$$

b) Find the magnetic field phasor, $\mathbf{H}_s$: With the Poynting vector in the positive x direction, a positive y component for $\mathbf{E}$ requires a positive z component for $\mathbf{H}$. Similarly, a positive z component for $\mathbf{E}$ requires a negative y component for $\mathbf{H}$. Therefore,

$$\underline{\mathbf{H}_s = \frac{10}{\eta_0}\left[\mathbf{a}_z - j\mathbf{a}_y\right]e^{-j25x}}$$

c) Describe the polarization of the wave: This is most clearly seen by first converting the given field to real instantaneous form:

$$\mathbf{E}(x,t) = \text{Re}\left\{\mathbf{E}_s e^{j\omega t}\right\} = 10\left[\cos(\omega t - 25x)\mathbf{a}_y - \sin(\omega t - 25x)\mathbf{a}_z\right]$$

At $x = 0$, this becomes,

$$\mathbf{E}(0,t) = 10\left[\cos(\omega t)\mathbf{a}_y - \sin(\omega t)\mathbf{a}_z\right]$$

With the wave traveling in the forward x direction, we recognize the polarization as left circular.

Q2:

I:

The region $z < 0$ is characterized by $\epsilon'_R = \mu_R = 1$ and $\epsilon''_R = 0$. The total $\mathbf{E}$ field here is given as the sum of the two uniform plane waves, $\mathbf{E}_s = 150e^{-j10z}\,\mathbf{a}_x + (50\angle 20°)e^{j10z}\,\mathbf{a}_x$ V/m.

a) What is the operating frequency? In free space, $\beta = k_0 = 10 = \omega/c = \omega/3 \times 10^8$. Thus, $\omega = 3 \times 10^9\ \text{s}^{-1}$, or $f = \omega/2\pi = \underline{4.7 \times 10^8 \text{ Hz}}$.

b) Specify the intrinsic impedance of the region $z > 0$ that would provide the appropriate reflected wave: Use

$$\Gamma = \frac{E_r}{E_{inc}} = \frac{50e^{j20°}}{150} = \frac{1}{3}e^{j20°} = 0.31 + j0.11 = \frac{\eta - \eta_0}{\eta + \eta_0}$$

Now

$$\eta = \eta_0\left(\frac{1+\Gamma}{1-\Gamma}\right) = 377\left(\frac{1 + 0.31 + j0.11}{1 - 0.31 - j0.31}\right) = \underline{691 + j177\ \Omega}$$

c) At what value of z ($-10\,\text{cm} < z < 0$) is the total electric field intensity a maximum amplitude? We found the phase of the reflection coefficient to be $\phi = 20° = .349$rad, and we use

$$z_{max} = \frac{-\phi}{2\beta} = \frac{-.349}{20} = -0.017\,\text{m} = \underline{-1.7\,\text{cm}}$$

II:

A uniform plane wave in air is normally-incident onto a lossless dielectric plate of thickness $\lambda/8$, and of intrinsic impedance $\eta = 260\ \Omega$. Determine the standing wave ratio in front of the plate. Also find the fraction of the incident power that is transmitted to the other side of the plate: With the a thickness of $\lambda/8$, we have $\beta d = \pi/4$, and so $\cos(\beta d) = \sin(\beta d) = 1\sqrt{2}$. The input impedance thus becomes

$$\eta_{in} = 260 \left[\frac{377 + j260}{260 + j377}\right] = 243 - j92\ \Omega$$

The reflection coefficient is then

$$\Gamma = \frac{(243 - j92) - 377}{(243 - j92) + 377} = -0.19 - j0.18 = 0.26\angle - 2.4\text{rad}$$

Therefore

$$s = \frac{1 + .26}{1 - .26} = \underline{1.7} \text{ and } 1 - |\Gamma|^2 = 1 - (.26)^2 = \underline{0.93}$$

Q3: $e^{-kpz} = e^{-1} = e^{-\delta z}$ (*from defination of penetration depth*)

$$\therefore k_0 pz = \delta z$$

$$\therefore \delta = k_0 p$$

$$\text{and from defination } \delta = \frac{1}{z}$$

$$\therefore \text{ for } \quad z \geq 10m \quad \delta \leq 0.1$$

$$\because \text{ water is a good conductor}$$

$$\therefore \text{attenuation coefficient } (p) \text{ can be approximated } p = \sqrt{30\sigma\lambda}$$

$$\therefore \delta = k_0 p = \frac{2\pi}{\lambda}\sqrt{30\sigma\lambda} = 2\pi\sqrt{\frac{30\sigma}{\lambda}} \leq 0.1$$

$$\therefore 2\pi\sqrt{\frac{30*4}{\lambda}} \leq 0.1$$

$$4\pi^2 \frac{120}{\lambda} \leq 0.01$$

$$4\pi^2 * 120 * 100 \leq \lambda$$

$$\therefore \lambda \geq 473.74\ km$$

$$\therefore\ f \leq 633.25\ Hz$$

Another solution: Using exact formula of P

$z > 10m$ & $\epsilon' = 80$ & $\sigma = 4$

Penetration depth $e^{-\delta z} = e^{-1} = e^{-k_0 p z}$

$$k_0 p z = 1 \Rightarrow z = \frac{1}{k_0 p} > 10m$$

So $$\frac{1}{\frac{2\pi}{\lambda} p} > 10$$

$$0.1 > \frac{2\pi}{\lambda} p$$

$\because$ $$n - jp = \sqrt{\epsilon' - j60\sigma\lambda}$$

after some manipulation

$$p = \sqrt{\frac{1}{2}\left[-\epsilon' + \sqrt{\epsilon'^2 + (60\sigma\lambda)^2}\right]}$$

So $$\frac{0.1}{2\pi} > \frac{1}{\lambda}\sqrt{\frac{1}{2}\left[-80 + \sqrt{6400 + (240\lambda)^2}\right]}$$

$$\left(\frac{0.1\lambda}{2\pi}\right)^2 > \frac{1}{2}\left[-80 + \sqrt{6400 + (240)^2\lambda^2}\right]$$

$$2\left(\frac{0.1\lambda}{2\pi}\right)^2 + 80 > \sqrt{6400 + (240)^2\lambda^2}$$

$$4\left(\frac{0.1\lambda}{2\pi}\right)^4 + 320\left(\frac{0.1\lambda}{2\pi}\right)^2 + 6400 > 6400 + (240)^2\lambda^2$$

Let's X=λ^2

$x > 9.24 \times 10^4$

$\lambda > 473.74 \times 10^3$

$c/f > 473.74 \times 10^3$

$633.02 > f$

$\Rightarrow f < 633.2$ let $f = 600$ HZ

Q4:

L.O.S. $f = 50\,MHz \Rightarrow \lambda = \frac{3\times10^8}{f} = 6m$

$h_1 = 20\,m$, $d = 15\,km$, flat earth $D = 1$

$F = ?$ $h_2 = ?$ max. received power $(\rho e^{j\phi} = -1)$

for max. received power

$$F_{max} = \left|1 - e^{-jk\Delta R}\right| = 2\left|\sin\left(\frac{k_o h_1 h_2}{d}\right)\right| = 2$$

max $\sin\left(\frac{k_o h_1 h_2}{d}\right) = 1$

in other word, receiver is ~~putting~~ placed at d_{max}

so $d_{max} = \frac{k_o h_1 h_2}{(2m+1)\frac{\pi}{2}} = \frac{4 h_1 h_2}{(2m+1)\lambda}$

let $m = 0$

$$d_{max} = \frac{4 h_1 h_2}{\lambda} = 15\,Km \text{ (Given)}$$

$$\text{so } h_2 = \frac{6 * 15\times10^3}{4 * 20} = 1.125\,km$$

II:

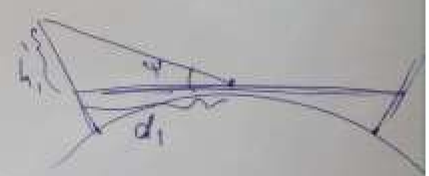
$a_e = 8500\ km$ (standard Troposphere)

$$d_0 = \sqrt{2a_e}\left(\sqrt{h_1} + \sqrt{h_2}\right) = 33.65\ km$$

$d < 0.8\, d_0$ (valid L.O.S.)

$$\text{so } d_1 \simeq \frac{h_1}{h_1+h_2} d = 7.14\ km$$

$$d_2 = d - d_1 = 2.85\ km$$



$$h_1' = h_1 - \Delta h_1 = 25 - \frac{d_1^2}{2a_e} = 22\ m$$

$$h_2' = h_2 - \Delta h_2 = 9.5\ m$$

$$D = \left[1 + \frac{2 d_1 d_2}{a_e(h_1' + h_2')}\right]^{-1/2} = 0.93$$

$$\tan\psi = \frac{h_1'}{d_1} \Rightarrow \psi = 0.176 \Rightarrow \rho e^{j\phi} \simeq -1$$

$$\text{so } |F| = \left|1 + (0.93)(-1)\, e^{-jk\Delta R}\right|$$

$$k\Delta R = \frac{2\pi}{\lambda}\left(\frac{2h_1' h_2'}{d}\right) = 150.4$$

$$|F| = 1.866 \Rightarrow \therefore E = E_d\, |F|$$

$$W_d = \frac{P_t G_t}{4\pi R^2} = \frac{E_d^2}{\eta} \quad \text{where } E_d : \text{Meansquare value}$$

$$\Rightarrow E_d = \sqrt{\frac{120\pi}{4\pi R^2} G_t P_t} = \frac{1}{R}\sqrt{30 G_t P_t}$$

$$E = \frac{1}{R}\sqrt{30 G_t P_t} * |F|$$

$$= \frac{1.866}{10k} * \sqrt{30 \times 50 \times 60} = 56\ mV/m$$

Q5:

I:

$d = 50\,km$, $f = 3\,GHz \Rightarrow \lambda = 0.1\,m$

h_1, h_2

midpath $h_1 = h_2$

$h_2 = h_1 = 0.6\,\rho_m + \Delta h_1$

$d_1 = d_2 = d/2$

$$\Delta h_1 = \frac{d_1^2}{2a_e} = \frac{0.25 * 50k}{2 * 8500\,k} = 36.75\ m$$

so $h_1 = 0.6\,\rho_m + 36.75$

$h_1 = h_2 = 57.98$ $\cancel{\#}$

II:

$d = 250\,km$

v.pol., $\epsilon_r = 10$, $\sigma = 0.01$

$P_t = 30\,kw$, $\lambda = 1200$, $G_t = 1.5$

$$b = \tan^{-1} \frac{\epsilon_r + 1}{x}$$

$x = 60\,\sigma\,\lambda = 720$

$\Rightarrow b = 0.875^\circ$

$$\Rightarrow p = \frac{\pi}{\lambda}\frac{R}{x} \cos b$$

$$= \frac{\pi \times 250k \times \cos(0.875^\circ)}{1200 \times 720} = 0.9089$$

3.14

$b < 5$ & $P < 4.5$ & vertical pol.

So $A_{v.p.} = e^{-0.43P} + 0.01P^2 = 0.676 + 8.26 \times 10^{-3} = 0.685$

$\therefore W = \frac{E_t^2}{2\eta}$ where E_t : total max field

$$W = \frac{G_t P_t}{4 \pi R^2} |2A|^2 = \frac{E_t^2}{2\eta}$$

$\Rightarrow E_t = 9.00 \times 10^{-3}$

if $E_{t\,r.m.s} = 6.36 \times 10^{-3}$

Q6:

$d = 40km$, $d_1 = d_2 = 20km$

(a) min. antenna height $= 0.6 \rho_m + \Delta h_1$

$$= 0.6 * 0.5 \sqrt{\lambda d} + \frac{d_1^2}{2a_e}$$

$h_1 = 18.9 + 23.5 = 42.5 \text{ m} = h_2$

(b) optimal height

$$h_1 = h_2 = \rho_m + \Delta h_1$$

$$= 31.6 + 23.5 = 55.1 \text{ m} = h_2$$

(c) for h_{opt} $\Rightarrow F = |1 + D|$

$$D = \left[1 + \left(\frac{2 d_1 d_2}{a_e (h'_1 + h'_2)}\right)\right]^{-\frac{1}{2}}$$

$h'_1 = h'_2 = \rho_m$

$$= \left[1 + \frac{2\left(d/_2\right)^2}{a_e\left(2\rho_m\right)^2} \right]^{-1/2} = 0.634$$

$\Rightarrow F = 1.634$

(e) $P_r = P_t * G_t G_r * |F|^2 * \left(\frac{\lambda}{4\pi R}\right)^2 = 5.2 \times 10^{-7}$ watt