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#### **Question** (1)

#### [20] Points

(a1) Define: Magnetic flux – magnetic field - magnetic field intensity- magnetic circuitmagnetic reluctance- magnetic flux density- magnetic permeability- magnetic hysteresis- eddy currents-iron losses- magnetic flux bunching- magnetic flux fringing- inrush current- BILautotransformer-current transformer-potential transformer- voltage regulation?

(b) A ferromagnetic core with a relative permeability of 2000 is shown in Figure 1. The dimensions are as shown in the diagram, and the depth of the core is 7cm. Because of fringing effects, the effective area of the air gaps is 5% larger than their physical size. The current in the coil is 1A. Determine the flux and flux density and magnetic reluctance in each part?



### Figure 1

### **Question** (2)

[20] Points

(a) Explain: interaction of magnetic fields- electro motive force (emf)- electromagnetic induced voltages- magneto motive force (mmf)- Polarity of the transformer?

(b) A 20KVA, 8000/480V, 60Hz, distribution transformer has a leakage impedance of (32+j45)  $\Omega$  in the high voltage winding and  $(0.05+j0.06)\Omega$  in the low voltage winding. The exciting branch impedance viewed from the high voltage side is  $(250K\Omega // j30K\Omega)$ . Assume the transformer is supplying rated load at 480V and 0.8 PF lagging. Draw the equivalent circuit and Determine the exciting current components, the losses, the efficiency, the per unit values, the voltage regulation?

## Question (3)

## [25] Points

(a) i-Write the common connections of the three phase transformers?

ii- Write the common conditions for parallel connections of transformers?

iii- Explain: three phase transformer and three phase transformer bank?

(b) Three single phase transformers A, B and C [50KVA, 2400/240V, 60Hz] are connected such that a Y- $\Delta$  three phase bank step down transformer has an output line voltage of 240V. Draw the circuit and determine:

i-The input line voltage? ii-The bank ratio and transformer ratio?

iii-The rated line and phase currents for high side and low side?

## **Question** (4)

## [25] Points

(a) Two single phase transformers A and B [60Hz, 100KVA] are connected to be in parallel to step down the input voltage. The respective no load voltage ratios and respective impedances as obtained from transformer nameplates are:

A has [2300/460V, R=1.36%, X=3.5%]

B has [2300/450V, R=1.4%, X=3.32%]

- i- Draw the circuit and determine the rated values?
- ii- Determine the secondary circulating current with no load?
- iii- State the results?

(b) Two single phase transformers A and B [150KVA, 4160/240V, 60Hz] are connected such that a V-V three phase bank step down transformers has an input line voltage of 4160V and an output line voltage of 240V. Draw the circuit and

i- Determine the bank ratio, the transformer ratio and bank KVA?

ii- Determine rated the line and phase currents for high side and low side?

Answer

#### Question (1)

#### [20] Points

(a1) Define: Magnetic flux – magnetic field - magnetic field intensity- magnetic circuitmagnetic reluctance- magnetic flux density- magnetic permeability- magnetic hysteresis- eddy currents-iron losses- magnetic flux bunching- magnetic flux fringing- inrush current- BILautotransformer-current transformer-potential transformer- voltage regulation?

A representative window-type CT is shown in Figure 3-18(a). The primary consists of a single power-line conductor looped through the window of a ferromagnetic toroid. T,he secondary has many turns of insulated wire wound around the toroid.

The alternating flux produced by current in the power-line conductor (primary in Figure 3-18(a)) induces current in the closed secondary circuit that is approximately proportional to the primary current. The actual CT ratio, as stamped on the nameplate, is generally expressed as a current ratio with respect to



Figure 3-18 Current transformer; (a) window type; (b) circuit connections.

# **Magnetic Field**

A magnetic field is a condition resulting from electric charges in motion. The magnetic field of a permanent magnet is attributed to the uncompensated spinning of electrons about their own axis within the atomic structure of the

material and to the parallel alignment of these electrons with







FIGURE 1-1 Direction of magnetic flux: (a) around a current-carrying conductor; (b) in a coil: (c) about a magnet.

# **1-3 Magnetic Circuit Defined**

Each magnetic circuit shown in Figure 1-2 is an arrangement of ferromagnetic materials called a core that forms a path to contain and guide the magnetic flux in a specific direction. The core shape shown in Figure I-2(a) is used in transformers. Figure I-2(b) shows the magnetic circuit of a simple two-pole motor; it includes a



FIGURE 1-2 Magnetic circuit: (a) for a transformer;

(b) for a simple two-pole motor.

stator core, a rotor core, and two air gaps. Note that the flux always takes the shortest path across an air gap.

# 1-14 Eddy Currents And Eddy-Current Losses

Eddy currents are circulating currents produced by transformer action in the iron cores of electrical apparatus. Figure 1-15(a) shows a block of iron that may be viewed as an infinite number of concentric shells or loops. The eddy voltages generated in these shells by a changing magnetic field are proportional to the rate of change of flux through the window of the respective shells. Thus, Expressed in terms of frequency and flux density, as00 obtained from Eq. (1-25),

$$E_e \alpha f. B_{max}$$

Slicing the core into man)' laminations and insulating one from the other will reduce the magnitude of the eddy currents by providing smaller paths, and hence lower eddy voltages. 'This is shown in Figure I-15(b). Laminated cores are made by stacking insulated steel stampings to the desired thickness or depth. Each lamination is insulated by a coating of insulating varnish or oxide on one or



Figure 1-15 currents in solid iron core; (b) laminated core.

both sides. Laminating the core results in much smaller shells, significantly reducing the heat losses in the iron.

The eddy-current loss, expended as heat power in the resistance of each shell, is proportional to the square of the eddy voyage.

Substituting Eq. (1-27) into Eq. (1-28), and applying a proportionalitj factor results in

$$P_e = k_e f^2 B_{\max}^2$$

where:

P<sub>e</sub> = eddy-current loss (W/unit mass)

f = frequency of flux wave (Hz)

B<sub>max</sub> = maximum value of flux density wave (T)

 $K_e$  = constant

The constant  $k_e$  is dependent on the lamination thickness, electrical resistivity, density and mass of the core material, and the units used.

## **Magnetic Field Intensity**

Magnetic field intensity, also called mmf gradient, is defined as the magnetomotive force per unit length of magnetic circuit or section of magnetic circuit, and is numerically equal to the ampere-turns applied to the magnetic circuit (section) divided by the effective length of the magnetic circuit (section). That is,

$$H = \frac{\mathcal{F}}{\ell} = \frac{N \cdot I}{\ell} \tag{1-2}$$

where:

H = magnetic field intensity (A-t/m)

 $\ell$  = mean length of the magnetic circuit, or section (m)

ℱ = mmf (A-t)

It should be noted that in a homogeneous magnetic circuit of cross section, the field intensity is the same at all points in the magnetic circuit. composite magnetic circuits, consisting of sections of different materials and/or] different cross-sectional areas, however, the magnetic field intensity differs from<sup>1</sup> section to section.

Magnetic field intensity has many useful application in magnetic circuit calculations. One specific application is calculating the magnetic-potential difference, also called magnetic drop or mmf drop, across a section of a magnetic circuit. The magnetic drop in ampere-tarns per meter of magnetic core length in a magnetic circuit is analogous to the voltage drop in volts per meter of conductor length in an electric circuit.

## **Flux Density**

The flux density is a measure of the concentration of lines of flux in a particular; section of a magnetic circuit. Expressed mathematically.

$$B = \frac{\Phi}{A}$$

 $\phi$  = flux, webers (Wb)

A = cross-sectional area (m<sup>2</sup>)

B = flux density (Wb/m<sup>2</sup>), or teslas (T)

# 1-4 Reluctance And The Magnetic Circuit Equation

A very useful equation that expresses the relationship between magnetic flux, t mmf. and the reiu; of the magnetic circuit is

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{N \cdot I}{\mathcal{R}}$$

where:

 $\phi$  = magnetic flux (Wb)

# =magnetomotive force (A-t)

 $\Re$  = reluctance of magnetic circuit (A-t/Wb)

Reluctance  $\Re$  is a measure of the opposition the magnetic circuit offers to the flux, and is analogous to resistance in an electric circuit. The reluctance of a magnetic circuit, or section of a magnetic circuit, is related to its length, cross-sectional area, and permeability. Solving Eq. (1-4) for  $\Re$  dividing numerator and denominator by  $\ell$  and rearranging terms,

$$\Re = \frac{N \cdot I}{\Phi} = \frac{N \cdot I/\ell}{\Phi/\ell} = \frac{H}{B \cdot A/\ell} = \frac{\ell}{(B/H) \cdot A}$$

Defining

$$\mu = \frac{B}{H}$$

Where

 $B = flux density (V/b/m^2) or teslas (T)$ 

H = magnetic field intensity (A-t/m)

 $\ell$  = mean length of magnetic circuit (m)

A = cross-sectional area  $(m^2)$ 

 $\mu$  = permeability of material (Wb/A-t-m)

Equation (1-6) applies to a homogeneous section of a magnetic circuit of uniform cross section.

## **Magnetic Permeability**

The ratio  $\mu$  = B/H is called magnetic permeability, and has different values for different degrees of magnetization of a specific magnetic core material.

# **1-5** Relative Permeability And Magnetization Curves

Relative permeability is the ratio of the permeability of a material to the permeability of free space; it is, in effect, a figure of merit that is very useful for comparing the magnetizability of different magnetic materials whose relative permeabilities are known. Expressed in equation form

$$\mu_r=\frac{\mu}{\mu_0}$$

where:

 $\mu_0$  = permeability of free space =  $4\pi 10^{-7}$  (Wb/A-t-m)

 $\mu_r$  = relative permeability, a dimensionless constant

 $\mu$  = permeability of material (Wb/A-t-m)

Representative graphs of Eq. (1-5) for some commonly used ferromagnetic materials are shown in Figure 1-3. The graphs, called B-H curves, magnetization curves, or saturation curves, are very useful in design, and in the analysis of machine and transformer behavior.

The four principal sections of a typical magnetization curve are illustrated in Figure 1-4. The curve is concave up for "low" values of magnetic field intensity, exhibits a somewhat (but not always) linear characteristic for "medium" field intensities, and then is concave down for "high" field intensities, eventually flattening to an almost horizontal line for "very high" intensities. The part of the curve that is concave down is known as the knee of the curve, and the "flattened" section is the saturation region. Magnetic saturation is complete when all of the magnetic domains of the material are oriented in the direction of the applied



FIGURE 1-3 Representative B-H curves for some commonly

used ferromagnetic materials.

magnetomotive force. Saturation begins at the start of the knee region and is essentially complete when the curve starts to flatten.

Depending on the specific application, the magnetic core of an apparatus may be operated in the linear region, the knee region, and/or the saturation region. For example, transformers and AC machines are operated in the linear region and lower end of the knee; self-excited DC generators and DC motors are operated in the upper end of the knee region, extending into the saturation region; separately excited DC generators are operated in the linear and tower end of the knee region. Magnetization curves supplied by manufacturers for specific electrical steel sheets or castings are usually plotted on semilog paper, and often include a curve of relative permeability vs. field intensity, as shown in Figure 1-5.'

The relationship between the relative permeability and the reluctance of a magnetic core is obtained by solving Eq. (1-7) for  $\mu$ , and then substituting into Eq. (1-6). The result is

$$\mathfrak{R}=\frac{\ell}{\mu A}=\frac{\ell}{\mu_{r}\mu_{0}A}$$



Magnetic Field Intensity (H, A-t/m)



the four principal sections.

Equation (1-8) indicates that the reluctance of a magnetic circuit is affected by the relative permeability of the material, which, as shown in Figure 1-5, is dependent on the magnetization, and hence is not constant. **1-7** Magnetic Hysteresis And Hysteresis Loss

If an alternating magnetomotive force is applied to a magnetic material, as shown in Figure I-8(a), and the flux density B plotted against the magnetic field intensity H, the resultant curve will indicate a lack of retraceability. This phenomenon, shown in Figure I-8(b), is called hysteresis, and the resultant curveis called an hysteresis loop.

Starting with an unmagnetized ferromagnetic core, point O on the curve, H = 0 and B = 0. Increasing the coil current in the positive direction increases the ampere-turns, and hence the magnetic field intensity.

$$H=\frac{NI}{\ell}$$

When the current reaches its maximum value, the flux density and magnetic field 1 intensity have their respective maximum values, and the curve is at point a; this initial trace of the curve, drawn with a broken line, is called the virgin section of the curve. As the current decreases, the curve follows a different path, and when



FIGURE 1-8 (a) Magnetic circuit with an alternating mmf:

(b) representative hysteresis loop.

the current is reduced to zero. H is reduced to zero, but the flux density in the core lags behind, holding at point b on the curve. The flux density at point b is the residual magnetism. This lagging of flux behind the magnetizing force is the hysteresis effect.

As the alternating current and associated magnetic field intensity increase in the negative direction, the residual magnetism decreases but remains positive until point c is reached, at which time the flux density in the core is zero. The negative field intensity required to force the residual magnetism to zero is called the coercive force, and is represented by line Oc on the W-axis. As the current continues its alternations, the plot of B vs. H follows points c-d-e-a-b-c on the hysteresis loop.

Magnetic hysteresis affects the rate of response of magnetic flux to a magnetizing force. In electrical apparatus such as transformers, in which the desired characteristic necessitates a quick and proportional response of flux to a change in mmf, with little residual magnetism, a high-grade silicon steel is used. Machines such as self-excited generators require steel that retains sufficient residual magnetism to permit the buildup of voltage. Stepper motors and some DC motors require permanent magnets with a very high magnetic retentivity (high hysteresis). Thus, the choice of magnetic materials is dictated by the application.

#### Magnetic Hysteresis Loss

If an alternating voltage is connected to the magnetizing coil, as shown in Figure I-8(a). the alternating magnetomotive force causes the magnetic domains to be constantly reoriented along the magnetizing axis. This molecular motion produces heat, and the harder the steal the greater the heat. The power loss due to hysteresis for a given type and volume of core material varies directly with the frequency and the nth power of the maximum value of the flux density wave.<sup>2</sup> Expressed mathematically,

$$\mathbf{P}_{\mathbf{h}} = \mathbf{K}_{\mathbf{h}} \cdot f \cdot \mathbf{B}_{\max}^{n}$$

where:

P<sub>h</sub> = hysteresis loss (W/unit mass of core)

f = frequency of flux wave (Hz)

 $\mathbf{B}_{\max}^{n}$  = maximum value of flux density wave (T)

 $k_h$  = constant

## 3-8 Transformer Inrush Current

When a switch is closed, connecting an AC source to an R-L series circuit (such as the equivalent series circuit of a transformer), the current will have a source-free response, called the transient component or inrush current, and a forced response called the steady-state component. Although the inrush component to a transformer decays rapidly, dropping to the normal no-load current within 5 to 10 cycles, it may exceed 25 times the full-load rating during the first half-cycle. This high inrush must be taken into consideration when selecting fuses and/or circuit breakers [12],[8].

The magnitude of the inrush depends on the magnitude and phase angle of the voltage wave at the instant the switch is closed, and the magnitude and direction of the residual flux in the iron'. If there is no residual magnetism, and) switch is closed at the instant the voltage wave has its maximum value, the current will be limited to the transformer no-load current, and there will he no inrush

Maximum inrush will occur if the switch is closed at the instant voltage wave is zero, and the build up of flux due to the build up of current is in a direction to reinforce the residual flux. If this occurs, saturation of the iron caused by the resultant high flux density, will reduce  $d\phi/dt$ , and this will deer the primary cemf, permitting a very high inrush current.

Closing the switch to a transformer is a random event. Hence, the ir current may be zero, very large, or some value in between. The inrush current also affected by the type and magnitude of the load connected to the secondary Inductive loads increase the inrush, whereas resistive loads and capacitive lo decrease the inrush.

(b) A ferromagnetic core with a relative permeability of 2000 is shown in Figure 1. The dimensions are as shown in the diagram, and the depth of the core is 7cm. Because of fringing effects, the effective area of the air gaps is 5% larger than their physical size. The current in the coil is 1A. Determine the flux and flux density and magnetic reluctance in each part?

Solution This core can be divided up into five regions. Let  $\Re_1$  be the reluctance of the left-hand portion of the core,  $\Re_2$  be the reluctance of the left-hand air gap,  $\Re_3$  be the reluctance of the right-hand portion of the core,  $\Re_4$  be the reluctance of the right-hand air gap, and  $\Re_5$  be the reluctance of the center leg of the core. Then the total reluctance of the core is

$$\begin{aligned} \Re_{\text{TOT}} &= \Re_5 + \frac{(\Re_1 + \Re_2)(\Re_3 + \Re_4)}{\Re_1 + \Re_2 + \Re_3 + \Re_4} \\ \Re_1 &= \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.11 \text{ m}}{(2000)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.07 \text{ m})} = 90.1 \text{ kA} \cdot t/\text{Wb} \\ \Re_2 &= \frac{l_2}{\mu_0 A_2} = \frac{0.0007 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.07 \text{ m})(1.05)} = 108.3 \text{ kA} \cdot t/\text{Wb} \\ \Re_3 &= \frac{l_3}{\mu_r \mu_0 A_3} = \frac{1.11 \text{ m}}{(2000)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.07 \text{ m})} = 90.1 \text{ kA} \cdot t/\text{Wb} \\ \Re_4 &= \frac{l_4}{\mu_0 A_4} = \frac{0.0005 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.07 \text{ m})(1.05)} = 77.3 \text{ kA} \cdot t/\text{Wb} \\ \Re_5 &= \frac{l_5}{\mu_r \mu_0 A_5} = \frac{0.37 \text{ m}}{(2000)(4\pi \times 10^{-7} \text{ H/m})(0.07 \text{ m})(0.07 \text{ m})} = 30.0 \text{ kA} \cdot t/\text{Wb} \end{aligned}$$

$$\Re_{\text{TOT}} = \Re_5 + \frac{(\Re_1 + \Re_2)(\Re_3 + \Re_4)}{\Re_1 + \Re_2 + \Re_3 + \Re_4} = 30.0 + \frac{(90.1 + 108.3)(90.1 + 77.3)}{90.1 + 108.3 + 90.1 + 77.3} = 120.8 \text{ kA} \cdot t/\text{Wb}$$

The total flux in the core is equal to the flux in the center leg:

$$\phi_{\text{center}} = \phi_{\text{TOT}} = \frac{\Im}{\Re_{\text{TOT}}} = \frac{(400 \text{ t})(1.0 \text{ A})}{120.8 \text{ kA} \cdot \text{t/Wb}} = 0.0033 \text{ Wb}$$

The fluxes in the left and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{\text{left}} = \frac{(\Re_3 + \Re_4)}{\Re_1 + \Re_2 + \Re_3 + \Re_4} \phi_{\text{TOT}} = \frac{(90.1 + 77.3)}{90.1 + 108.3 + 90.1 + 77.3} (0.0033 \text{ Wb}) = 0.00193 \text{ Wb}$$
$$\phi_{\text{right}} = \frac{(\Re_1 + \Re_2)}{\Re_1 + \Re_2 + \Re_3 + \Re_4} \phi_{\text{TOT}} = \frac{(90.1 + 108.3)}{90.1 + 108.3 + 90.1 + 77.3} (0.0033 \text{ Wb}) = 0.00229 \text{ Wb}$$

The flux density in the air gaps can be determined from the equation  $\phi = BA$ :

$$B_{\text{left}} = \frac{\phi_{\text{left}}}{A_{\text{eff}}} = \frac{0.00193 \text{ Wb}}{(0.07 \text{ cm})(0.07 \text{ cm})(1.05)} = 0.375 \text{ T}$$
$$B_{\text{right}} = \frac{\phi_{\text{right}}}{A_{\text{eff}}} = \frac{0.00229 \text{ Wb}}{(0.07 \text{ cm})(0.07 \text{ cm})(1.05)} = 0.445 \text{ T}$$

**Question** (2)

#### [20] Points

(a) Explain: interaction of magnetic fields- electro motive force (emf)- electromagnetic induced voltages- magneto motive force (mmf)- Polarity of the transformer?

# **Interaction Of Magnetic Fields (Motor Action)**

- (b) When two or more sources of magnetic fields are arranged so that their fluxes, or a component of their fluxes, are parallel within a common region, a mechanical force will be produced that tends to either force the sources of flux together or to force them apart. A force of repulsion will occur if the two magnetic sources have components of flux that are parallel and in the same direction; this will be indicated by a net increase in flux called "flux bunching" in the common region. A force of attraction will occur if the respective fluxes have components that are parallel and in opposite directions; this will be indicated by a net increase in flux called "flux bunching" in the common region.
- (c)

# (d) Forces on Adjacent Conductors

- (e) The interaction of magnetic fields of adjacent current-carrying conductors produces mechanical forces that tend to bring together or separate the two conductors. If the currents in-adjacent conductors are in opposite directions, as shown in Figure I-9(a), the respective components of flux in the common region will be in the same direction, and as indicated by flux bunching, a separating force will be produced on the conductors. If the currents in adjacent conductors are in the same direction, as shown in Figure I-9(b), the respective components of flux in Figure I-9(b), the respective components of flux in the currents in adjacent conductors are in the same direction, as shown in Figure I-9(b), the respective components of flux in the common region will be in opposite directions, and the net reduction in flux indicates a force of attraction.
- (f) Under severe short-circuit conditions, the forces between adjacent conductors can be high enough to physically crush the insulation of transformers,

motors, and generators, bend bus bars, tear switchboards apart, and cause switches and circuit breakers to come apart with explosive violence. Thus, in those applications where the available short-circuit current is of a magnitude that would cause destruction of apparatus if a fault occurred, special currentlimiting devices, as well as mechanical bracing and conductor support must be installed [1], [2].

(g)





(j)

# (k)1-9 Elementary Two-Pole Motor

Figure 1-10 shows a rotor core, containing two insulated conductors in rotor slots, and the rotor centered between the poles of a stationary magnet (called the

stator). The + mark on the end of conductor A is the tail end of an arrow that represents the direction of current in conductor A. The dot in the center of conductor B is the point of an arrow indicating the direction of current in conductor B. The direction of flux around each

# **1-11 Electromagnetically Induced Voltages (Generator Action)**

The magnitude of the voltage induced in a coil by electromagnetic induction is directly proportional to the number of series-connected turns in the coil, and to the rate of change of flux through its window. This relationship, known as Faraday's law, is expressed mathematically as

$$\mathbf{e} = \mathbf{N} \frac{\mathbf{d}\boldsymbol{\phi}}{\mathbf{d}t}$$

where:

e = induced voltage (electromotive force, emf) (V)

N = number of series-connected turns

 $d\phi/dt$  = rate of change of flux through window (Wb/s)

The basic Faraday relationship expressed in Eq. (1-15) is often converted by mathematical manipulation to other forms for solution of specific groups of problems.

Electroniagnetically induced voltages are generated by relative motion or transformer action. Voltages generated by transformer action are due to llux varying with time through the window of a stationary coil. Voltages generated by relative motion involve a moving coil and a stationary magnet, or a moving magnet and a stationary coil. Voltages caused by relative motion are called speed voltages or "flux cutting" voltages.

In accordance with Lenz's law, the voltage, current, and associated flux, generated by transformer action, or relative motion between a conductor and a magnetic field, will always be induced in a direction to oppose the action that caused it.<sup>3</sup> In a transformer, the flux due to current generated in a transformer coil will be in a direction to oppose the change in flux that caused it.

In the case of a conductor driven by an applied force, the flux due to current generated in the conductor will set up a counlerfbrce in opposition to the applied force. In a rotating machine, the flux due to generated current in the conductors will set up a countertorque (motor action) in opposition to the driving torque of the prime mover. In fact, as will be shown in subsequent chapters, all generators may be operated as motors and all motors may be operated, as generators.

Speed Voltages and the BLV Rule

A closed loop consisting of two conductors X and Y, and a set of conducting rails, is situated within a uniform magnetic field, as shown in Figure I-13(a); conductor Y is clamped and conductor X is moving to the right at velocity v-meters per second. The window in Figure I-13(a) is the area enclosed by conductor X, conductor Y, and the-conducting rails. As conductor X moves to the right, the window area increases, causing the flux through the window to increase with time, inducing a voltage in the loop.



FIGURE 1-13 Closed loop consisting of two conductors and a set of conducting rails; (b) direction of emf and current caused by conductor moving to the right; (c) equivalent circuit, both conductors moving in the same direction; (d) equivalent circuit, conductors moving in opposite directions.

Expressing the flux in terms of the flux density and the area of the window,

$$\phi = B.A$$

Taking the derivative with respect to time,

$$\frac{d\phi}{dt} = B \cdot \frac{dA}{dt}$$

Substituting into Eq. (1-15)

$$e = N \cdot B \, \frac{dA}{dt}$$

From Figure I-13(a), the increment increase in window area, as conductor X moves to the right, may be expressed in terms of length  $\ell$  and an increment increase in distance (ds) along the rails. That is,

$$dA = \ell ds$$

Substituting into Eq. (1-16), and noting that N = 1 for a single loop,

$$\mathbf{e} = \mathbf{B} \cdot \ell \cdot \frac{\mathbf{d}\mathbf{s}}{\mathbf{d}\mathbf{t}}$$

Since ds/dt represents the velocity of the conductor, Eq. (I-18) may be rewritten as

where:

e = induced voltage (V)

B = flax density of field (T)

 $\ell$  = effective length of conductor (m)

v = velocity of conductor (m/s)

Note: For the loop formed by the rod and rails in Figure I-13(a), conductor X is the only moving conductor.

Since the emf was generated by an applied force driving conductor X to the right, the induced voltage and associated current will be in a direction to develop a counterforce. For this to happen, flux bunching must occur on **Magnetomotive** 

# Force

The ampere-turns (A-t) of the respective coils in Figure 1-2 represent the driving force, called magnetomotive force or mmf, that causes a magnetic field to appear in the corresponding magnetic circuits. Expressed in equation form,

**ℱ** = ℕ-I (1-1)

where:

# = magnetomotive force (mmf) in ampere-turns (A-t)N = number of turns in coil

I = current in coil (A)

## (a)2-2 Construction Of Power And Distribution Transformers

(b) The two basic types of transformer construction used for power and distribution applications are shown in Figure 2-1. Note that the high-voltage coils are wound with a greater number of turns of smaller cross-section conductor than the low-voltage coils. The core type, shown in Figure 2-I(a), has primary and secondary coils wound on different legs, and the shell type, shown in Figure 2-I(b), has both coils wound on the same leg. The wider spacing between primary and secondary in the core-type transformer gives it an advantage in high-voltage applications. The shell type, however, has the advantage of less leakage flux.

(c)





(g)

Transformer core material is made of nonaging, cold-rolled, high-pern ability silicon steel laminations, and each lamination is insulated with a varnish < oxide coating io reduce eddy currents. The coils are wound with insulated alun num conductor or insulated copper conductor, defending on design considerations. Cooling is provided by air convection, forced air, insulating liquids, or gas. **2-11** 

# **Voltage Regulation**

The effects of leakage flux and winding resistance in a transformer cause internal voltage drops that result in different output voltages for different loads. The difference between the output voltage at no load and the output voltage at rated load, divided by the output voltage at rated load, is called the voltage regulation of the transformer, and is commonly used as a figure of merit when comparing transformers. Expressed mathematically,

$$\operatorname{reg} = \frac{E - V_{\operatorname{rated}}}{V_{\operatorname{rated}}}$$
(2-44)

where:

E = voltmeter reading at the output terminals when no load is connected to the transformer

 $V_{rated}$  = voltmeter reading at the output terminals when the transformer is supplying rated apparent power

The inrush current to a transformer during the first few cycles depends on the instantaneous value of the voltage wave at the moment the switch or breaker is closed. Since the inrush current may exceed 25 times rated current, it is essential that this phenomenon be understood and taken into consideration when selecting fuses or circuit breakers.

## **4** Autotransformers

An autotransformer. shown in Figure ?-2fa), uses a single coil with one or more taps to provide transformer action; the input/output connections for operation in the step-down mode are shown in Figure 3-2(b). where

 $N_{HS}$  = number of turns in the high side

N<sub>LS</sub> = number of turns embraced by the low side

# **6** Parallel Operation Of Transformers

When increases in industrial or utility loads approach the full-load rating of a transformer, another transformer of similar rating is generally paralleled with the first, and the load shared between them. For optimum conditions when operating in parallel, however, transformers should have the same turns ratio, identical impedances, and identical ratios of resistance to reactance. Transformers with different turns ratios will have circulating currents in the paralleled loop formed by the transformer secondaries, and transformers with unlike impedances will divide the load in the inverse ratio of their impedances.

(b) A 20KVA, 8000/480V, 60Hz, distribution transformer has a leakage impedance of (32+j45)  $\Omega$  in the high voltage winding and  $(0.05+j0.06)\Omega$  in the low voltage winding. The exciting branch impedance viewed from the high voltage side is  $(250K\Omega // j30K\Omega)$ . Assume the transformer is supplying rated load at 480V and 0.8 PF lagging. Draw the equivalent circuit and Determine the exciting current components, the losses, the efficiency, the per unit values, the voltage regulation?

(a) The turns ratio of this transformer is a = 8000/480 = 16.67. Therefore, the secondary impedances referred to the primary side are

$$R_s' = a^2 R_s = (16.67)^2 (0.05 \ \Omega) = 13.9 \ \Omega$$
  
 $X_s' = a^2 X_s = (16.67)^2 (0.06 \ \Omega) = 16.7 \ \Omega$ 

The resulting equivalent circuit is



(b) The rated kVA of the transformer is 20 kVA, and the rated voltage on the primary side is 8000 V, so the rated current in the primary side is 20 kVA/8000 V = 2.5 A. Therefore, the base impedance on the primary side is

$$Z_{\rm base} = \frac{V_{\rm base}}{I_{\rm base}} = \frac{8000 \, \rm V}{2.5 \, \rm A} = 3200 \, \Omega$$

Since  $Z_{pu} = Z_{actual} / Z_{base}$ , the resulting per-unit equivalent circuit is as shown below:



(c) To simplify the calculations, use the simplified equivalent circuit referred to the primary side of the transformer:



The secondary current in this transformer is

$$I_s = \frac{20 \text{ kVA}}{480 \text{ V}} \angle -36.87^\circ \text{ A} = 41.67 \angle -36.87^\circ \text{ A}$$

The secondary current referred to the primary side is

$$\mathbf{I}_{s}' = \frac{\mathbf{I}_{s}}{a} = \frac{41.67 \angle -36.87^{\circ} \text{ A}}{16.67} = 2.50 \angle -36.87^{\circ} \text{ A}$$

Therefore, the primary voltage on the transformer is

$$\mathbf{V}_{p} = \mathbf{V}_{S}' + (R_{EQ} + jX_{EQ})\mathbf{I}_{S}'$$
  
$$\mathbf{V}_{p} = 8000 \angle 0^{\circ} \text{ V} + (45.9 + j61.7)(2.50 \angle -36.87^{\circ} \text{ A}) = 8185 \angle 0.38^{\circ} \text{ V}$$

The voltage regulation of the transformer under these conditions is

$$VR = \frac{8185 - 8000}{8000} \times 100\% = 2.31\%$$

(d) Under the conditions of part (c), the transformer's output power copper losses and core losses are:

$$P_{\text{OUT}} = S \cos \theta = (20 \text{ kVA})(0.8) = 16 \text{ kW}$$
$$P_{\text{CU}} = (I_s')^2 R_{\text{EQ}} = (2.5)^2 (45.9) = 287 \text{ W}$$
$$P_{\text{core}} = \frac{V_s'^2}{R_c} = \frac{8185^2}{250,000} = 268 \text{ W}$$

The efficiency of this transformer is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}}} \times 100\% = \frac{16,000}{16,000 + 287 + 268} \times 100\% = 96.6\%$$

### **Question (3)**

### [25] Points

(a) i-Write the common connections of the three phase transformers?

1-	Y	Y	2-	YΔ	3-∆	Y	4-22	5-Y	YΔ	6-VV
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# **3-10 Three-Phase Connections Of Single-Phase Transformers**

Most AC power is generated and distributed as 3-phase. The voltage is raised or lowered with 3-phase transformers, or with a bank of single-phase transformers connected in 3-phase arrangements, as shown in Figure 3--12. The current and voltage relationships between phase and line values for a wye connection are<sup>5</sup>

$$V_{\text{line}} = \sqrt{3} \cdot V_{\text{phase}}$$
  $I_{\text{line}} = I_{\text{phase}}$ 

The current and voltage relationships between phase and line values for a delta connection are

$$I_{\text{line}} = \sqrt{3} \cdot I_{\text{phase}} \qquad V_{\text{line}} = V_{\text{phase}}$$



FIGURE 3-12 Three-phase connections of single-phase transformers.

### **Delta-Delta and V-V Banks**

The delta-delta bank, shown in Figure 3-12(b) and in Figure (3-13a). has advantage of being able to operate continuously with one of the three transformers disconnected from the circuit. This open-delta connection, also called a V-V connection, provides a convenient means for inspection, maintenance, testing and replacing of transformers one at a time, with only a brief power interruption. The open-delta connection is also used to provide 3phase service in applications



FIGURE 3-13 (a) Delta-delta bank; (b) phasor diagram for (a); (c) V-V bank; (d) phasor diagram for (c).

ii- Write the common conditions for parallel connections of transformers? **3-12** Beware The **30°** Phase Shift When Paralleling Three-Phase Transformer Banks

There is an angular displacement, called phase shift, between the corresponding primary and secondary line voltages in the Y- $\Delta$  bank and in the  $\Delta$ -Y bank (as shown in Figure 3-12), with the low voltage lagging the high voltage by 30°. There is no angular displacement between corresponding primary and secondary voltages in a Y-Y bank.  $\Delta$ - $\Delta$  bank, or a V-V bank. Because of the phase inherent in Y- $\Delta$  and  $\Delta$ -Y banks, they must not be paralleled with Y-Y,  $\Delta$ - $\Delta$ , V-V banks; to do so would cause large circulating currents and severe overheating of the windings [5].

Only banks with the same phase shift should be operated in parallel. It should be noted that the bank ratio (ratio of line voltages) for Y-Y,  $\Delta$ - $\Delta$ . or V-V banks is equal to the respective turns ratios. This may be deduced from Figure 3-12(a). (b). and (e).

# **Parallel Operation Of Transformers**

When increases in industrial or utility loads approach the full-load rating of a transformer, another transformer of similar rating is generally paralleled with the first, and the load shared between them. For optimum conditions when operating in parallel, however, transformers should have the same turns ratio, identical impedances, and identical ratios of resistance to reactance. Transformers with different turns ratios will have circulating currents in the paralleled loop formed by the transformer secondaries, and transformers with unlike impedances will divide the load in the inverse ratio of their impedances.

iv- Explain: three phase transformer and three phase transformer bank? Three phase transformer using one core for three phase





FIGURE 3-14 Basic construction of ?-phase transformers: (a) shell type;

(b) core type.

Three phase transformer bank using three single phase transformers connected to obtain one three phase transformer

(b) Three single phase transformers A, B and C [50KVA, 2400/240V, 60Hz] are connected such that a Y- $\Delta$  three phase bank step down transformer has an output line voltage of 240V. Draw the circuit and determine:

i-The input line voltage? ii-The bank ratio and transformer ratio?

iii-The rated line and phase currents for high side and low side?

The input line voltage =2400\*3^0.5=4157V

The bank ratio= $V_{line}/V_{line}=(4157)/240=17.32$ 

The trans. ratio=V<sub>phase</sub>/V<sub>phase</sub>=2400/240=10

Rated line current Y=3\*50000/(4157\*3^0.5)=20.833A



Assume that the phase voltages on the primary side are given by  $\mathbf{V}_A = V_{\phi P} \angle 0^\circ$   $\mathbf{V}_B = V_{\phi P} \angle -120^\circ$   $\mathbf{V}_C = V_{\phi P} \angle 120^\circ$ 

Then the phase voltages on the secondary side are given by

$$\mathbf{V}_{A} = V_{\phi s} \angle 0^{\circ} \qquad \mathbf{V}_{B} = V_{\phi s} \angle -120^{\circ} \qquad \mathbf{V}_{C} = V_{\phi s} \angle 120^{\circ}$$

where  $V_{\phi S} = V_{\phi P} / a$ . Since this is a Y- $\Delta$  transformer bank, the line voltage  $V_{ab}$  on the primary side is

$$\mathbf{V}_{ab} = \mathbf{V}_A - \mathbf{V}_B = V_{\phi p} \angle 0^\circ - V_{\phi p} \angle -120^\circ = \sqrt{3} V_{\phi p} \angle 30^\circ$$

and the voltage  $\mathbf{V}_{a'b'} = \mathbf{V}_{A} = V_{\phi\delta} \angle 0^{\circ}$ . Note that the line voltage on the secondary side lags the line voltage on the primary side by 30°.

Assume that the phase voltages on the primary side are given by

$$\mathbf{V}_A = V_{\phi p} \angle 0^\circ \qquad \mathbf{V}_B = V_{\phi p} \angle -120^\circ \qquad \mathbf{V}_C = V_{\phi p} \angle 120^\circ$$

Then the phase voltages on the secondary side are given by

$$\mathbf{V}_{A} = \mathbf{V}_{\phi S} \angle 0^{\circ} \qquad \mathbf{V}_{B} = \mathbf{V}_{\phi S} \angle -120^{\circ} \qquad \mathbf{V}_{C} = \mathbf{V}_{\phi S} \angle 120^{\circ}$$

where  $V_{\phi\delta} = V_{\phi p} / a$ . Since this is a  $\Delta$ -Y transformer bank, the line voltage  $V_{ab}$  on the primary side is just equal to  $V_A = V_{\phi p} \angle 0^\circ$ . The line voltage on the secondary side is given by

 $\mathbf{V}_{ab'} = \mathbf{V}_A - \mathbf{V}_C = V_{\phi p} \angle 0^\circ - V_{\phi p} \angle 120^\circ = \sqrt{3} V_{\phi p} \angle -30^\circ$ 

### **Question (4)**

#### [25] Points

(a) Two single phase transformers A and B [60Hz, 100KVA] are connected to be in parallel to step down the input voltage. The respective no load voltage ratios and respective impedances as obtained from transformer nameplates are:

Transformer	Voltage ratio	%R	%X
А	2300-460	1.36	3.5
В	2300-450	1.4	3.32

v- Draw the circuit and determine the rated values?

vi- Determine the secondary circulating current with no load?

vii- State the results?

The rated low side current I<sub>A</sub>=100000/460=217.39A, rated Z<sub>A</sub>=460/217.39=2.12ohm

R<sub>Aequ</sub>=0.0136\*2.12=0.0288320hm, X<sub>Aequ</sub>=0.035\*2.12=0.07420hm

The rated low side current  $I_B$ =100000/450=222.22A, rated  $Z_B$ =450/222.2=2.025ohm

 $R_{Bequ}=0.014*2.025=0.02835$  ohm,  $X_{Bequ}=0.0332*2.025=0.06723$  ohm

 $R_{B+Aequ} = 0.02835 + 0.028832 = 0.0572 \text{ ohm}, X_{B+Aequ} = 0.06723 + 0.0742 = 0.14143 \text{ ohm}$ 

 $Z_{A+B}=R_{B+Aequ}+j X_{B+Aequ}=0.0572+j0.14143=0.1524L67.97ohm$ 

 $I_{cir} = (E_A - E_B)/(Z_A + Z_B) = (460 \text{L}0 - 450 \text{L}0)/0.1524 \text{L}67.97 = 65.62 \text{L} - 67.97 \text{A}$ 

 $I_{cir}=65.62/217.39=30.2\%, 10/460=2.2\%$ 

2.2% in secondary voltage causes 30.2% of the rated current circulating in the secondaries



(b) Two single phase transformers A and B [150KVA, 4160/240V, 60Hz] are connected such that a V-V three phase bank step down transformers has an input line voltage of 4160V and an output line voltage of 240V. Draw the circuit and

- i- Determine the bank ratio, the transformer ratio and bank KVA?
- ii- Determine rated the line and phase currents for high side and low side?

The input line voltage =input phase voltage =4160V

The output line voltage =output phase voltage =240V

The bank ratio = The trans. ratio = $V_{line}/V_{line}=(4160)/240=17.33$ 

bank KVA ==(3^0.5) \*150=259.8 KVA=[(3^0.5)  $V_{\text{line}*}I_{\text{line}}]_{\Delta\Delta}/(3^0.5)$ 

Rated high side vv line current = Rated phase current = $259800/((3^{0.5}) 4160)$ =36A

Rated low side vv line current = Rated phase current =259800/((3^0.5) 240))=624.98A

