$$
\begin{aligned}
& \text { د شوقي حامد عرفه كلية الهندسة ببنها }
\end{aligned}
$$

| Benha University | Time: 3hour |
| :--- | :--- |
| Benha Faculty of Engineering | Forth Year 2014/2015 |
| Subject: Power Electronics (E462) | Elect. Eng. Dept. |

## Solve \& draw as much as you can (questions in two pages)

## Question (1)

## [12] Points

a- Explain in details the power and energy losses in a real semi-conductor switch?
b-Explain in details a single phase bridge inverter?
c-Define harmonic factor and total harmonic distortion?
d-Explain in details two methods of PWM?

## Question (2)

## [12] Points

a- Explain in details a dc-dc chopper?
b- A dc-dc chopper class A has a resistive load of $10 \Omega$ and the input dc voltage $\mathrm{Vs}=220 \mathrm{~V}$. The switch voltage drop is 2 V and the chopping frequency is 1 Khz , if the duty cycle is $50 \%$ Determine (i) the average output voltage (ii) the rms output voltage (iii) the output power?

## Question (3)

## [12] Points

a- Explain in details a single phase $\mathrm{AC} / \mathrm{AC}$ voltage controller?
b- A single-phase full wave ac voltage controller has a resistive load of $10 \Omega$ and supply voltage of $120 \mathrm{~V}, 60 \mathrm{~Hz}$. The delay angles of the two thyristors are equal to $90^{\circ}$. (i) Draw the power circuit and its associated voltage and current waveforms. (ii) Determine the rms values of output voltage and current, (iii) the average value of the thyristor current?

## P.T.O.

## [12] Points

a- Explain in details a single phase transformer tap-changer?
b- A single phase transformer tap-changer has a primary voltage of $240 \mathrm{~V}, 60 \mathrm{~Hz}$. The secondary voltages are $\mathrm{V}_{1}=120 \mathrm{~V}$ and $\mathrm{V}_{2}=120 \mathrm{~V}$. If the load resistance is $10 \Omega$ and firing angles of thyristors T1 and T2 are $98^{\circ}$ and $\pi+98^{\circ}$ respectively. Determine (i) the load voltage, (ii) the rms current of the four thyristors (iii) the rms current of the secondary windings?

## Question (5)

## [12] Points

a- Explain in details a three phase bridge inverter?
b- Design a three-phase PWM inverter that drives a 30hp, variable speed, 3-phase, 12-pole, $460 \mathrm{~V}, 60 \mathrm{~Hz}$ induction motor. The DC supply is 300 V and $\mathrm{m}_{\mathrm{a}}=0.8$ and $\mathrm{m}_{\mathrm{f}}=15$. Draw a block diagram of the system and use the following table to calculate the fundamental frequency line to line voltage and some dominant harmonics of line to line voltages.

|  | $\mathrm{m}_{\mathrm{a}}$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| h | 0.2 | 0.4 | 0.6 | 0.8 | 1 |  |
| 1 | 0.122 | 0.245 | 0.367 | 0.490 | 0.612 |  |
| $\mathrm{~m}_{\mathrm{f}} \pm 2$ | 0.010 | 0.037 | 0.080 | 0.135 | 0.195 |  |
| $2 \mathrm{~m}_{\mathrm{f}} \pm 1$ | 0.116 | 0.20 | 0.227 | 0.192 | 0.111 |  |
| $3 \mathrm{~m}_{\mathrm{f}} \pm 2$ | 0.027 | 0.085 | 0.124 | 0.108 | 0.038 |  |

## Question (1)

## [12] Points

a- Explain in details the power and energy losses in a real semi-conductor switch?

### 5.4 Power losses from manufacturers' data sheets

The total power dissipation $P_{d}$ is the sum of the switching transition loss $P_{s}$, the onconduction loss $P_{d}$, drive input device loss $P_{G}$ and the off-state leakage loss $P_{t}$.
The average total power loss is given by

$$
\begin{equation*}
P_{d}=f_{s} \int_{0}^{1 / d} v(t) i(t) d t \tag{W}
\end{equation*}
$$

where $f_{s}$ is the switching frequency and $v(t)$ and $i(t)$ are the device instantaneous voltage and current over one complete cycle of period $1 / f_{s}$. The usual technique for determining total power loss is to evaluate and sum together each of the individual average power loss components.

### 5.4.1 Switching transition power loss, $P_{s}$

Figure 5.7 shows typical power device voltage-current switching waveforms. Normally an exact solution is not required and an approximation based on straight line switching intervals is usually adequate.
For a resistive load, as derived in chapter 6

$$
\begin{equation*}
P_{s}=\frac{l}{6} V_{s} I_{m} \tau f_{s} \tag{W}
\end{equation*}
$$

and for an inductive load, as derived in chapter 6

$$
\begin{equation*}
P_{s}=1 / 2 V_{s} I_{m} \tau f_{s} \tag{W}
\end{equation*}
$$

where $\tau$ is the period of the switching interval (both on and off), and $V_{s}$ and $I_{m}$ are the maximum voltage and current levels as shown in figure 5.7. Switching losses occur at both turn-on and turn-off.


Figure 5.7. Typical voltage and current at turn-off switching transition for: (a) an inductive load and (b) a resistive load. Current and voltage are interchanged at turn-on.

### 5.4.2 Off-state leakage power loss, $P_{t}$

During the switched-off period, a small, exponentially temperature dependent current $I_{t}$, will flow through the switch. The loss due to this leakage current is

$$
\begin{equation*}
P_{\ell}=I_{t} V_{t}(1-\delta) \tag{W}
\end{equation*}
$$

where $\delta$ is the on-time duty cycle of the switch. Normally $P_{t}$ is only a small part of the total loss so that the error in neglecting $P_{\ell}$ is not usually significant.

### 5.4.3 Conduction power loss, $P_{c}$

The average conduction power loss under a steady-state current condition is given by

$$
\begin{equation*}
P_{c}=\delta I_{o n} V_{o n} \tag{W}
\end{equation*}
$$

although equation (5.21) is valid in the general case when the integration is performed over the interval corresponding to $\delta$.
The conduction loss for the MOSFET is usually expressed in terms of its on-state resistance (equations (3.14) and (4.12))

$$
\begin{align*}
P_{c} & =\delta I_{d(m m)}^{2} R_{d(m)} \\
& =\delta I_{d(m u))}^{2} R_{d(m)}\left(25^{\circ} \mathrm{C}\right)\left\{1+\frac{\alpha}{100}\right\}^{T_{j}-25^{\circ} \mathrm{C}} \tag{5.26}
\end{align*}
$$

where $\alpha$ is the temperature coefficient of the on-state resistance, which is positive. A linear resistance approximation of equation (5.26) is quite accurate above $25^{\circ} \mathrm{C}$ if $\alpha$ is small, such that $P_{c}$ can be approximated by

$$
\begin{equation*}
P_{c} \approx \delta I_{d(m s)}^{2} R_{d(m a)}\left(25^{\circ} \mathrm{C}\right)\left\{1+\alpha\left(T_{j}-25^{\circ} \mathrm{C}\right)\right\} \tag{W}
\end{equation*}
$$

### 5.4.4 Drive input device power loss, $P_{G}$

A portion of the drive power is dissipated in the controlling junction or, in the case of the MOSFET, in the internal gate resistance. Usually more power is dissipated in the actual external drive circuit resistance. Drive input loss is normally small and insignificant compared with other losses, and can usually be ignored.
Two possible exceptions are:

- One notable exception is in the case of the power thyristor, where continuous gate drive is used to avoid loss of latching or when the holding current is high. The holding current can be $3 \%$ of the anode current thus the gate to cathode junction loss can be included in the total loss calculation for better accuracy. Thus, for a gate junction voltage $V_{G C}$ the gate losses are given by

$$
\begin{equation*}
P_{g}=\delta I_{G} V_{G C} \tag{5.28}
\end{equation*}
$$

The recovery loss of the gate commutated thyristor (GCT) cathode junction can be included since it is significant because the full anode current is extracted from the gate, thus is involved in recovery of the cathode junction.

- A second exception is the MOSFET and IGBT at high switching frequencies, $>50 \mathrm{kHz}$, when the loss in the device, associated with providing the gate charge $Q_{T}$ is given by equation (4.35):

$$
\begin{equation*}
P_{G}\left(R_{\text {int }}\right)=V_{z z} Q_{T} f_{s} \frac{R_{G \text { iint }}}{R_{G \text { int }}+R_{G \text { Gast }}} \tag{W}
\end{equation*}
$$

### 5.5 Heat-sinking design cases

Heat-sink design is essentially the same for all power devices, but the method of determining power loss varies significantly from device type to device type. The information given in data sheets, in conjunction with the appropriate equation in table 5.2 , allows the designer to calculate power semiconductor thermal rating for a variety of conditions.

### 5.5.1 Heat-sinking for diodes and thyristors

At low switching frequencies ( $<100 \mathrm{~Hz}$ ), switching loss can be ignored. In the case of rectifying diodes or converter-grade thyristors, 50 to 60 Hz , switching loss can usually be ignored. Fast-recovery power diodes switching at less than 500 Hz can also have switching losses neglected at low $V A$ levels.

### 5.5.1i - Low-frequency switching

At a given current level $I_{F}$ and on-time duty cycle $\delta$, on-state power loss can be read directly from the manufacturers' data. Figure 5.8 a illustrates loss for square-wave power pulses, while figure 5.8 b illustrates loss in the case of half-wave sinusoidal current. Figure 5.8 b gives energy loss per cycle, which may be converted to power when multiplied by the sinusoidal pulse frequency.
Thyristor loss due to the current waveform initial rate of rise of current, $d i / d t$, can be incorporated and its contribution is added into the manufacturers' conduction loss data for a given device type.

### 5.5.1ii - High-frequency switching

At frequencies greater than about 100 Hz , fast-recovery diodes are normally employed and at about 500 Hz , switching losses must be added to the on-state conduction loss. Diode turn-off loss is usually more significant than turn-on loss. Manufacturers provide maximum reverse recovery charge, $Q_{R}$, characteristics as shown in figure 5.9. The reverse recovery charge is a linear function of temperature and between the given junction temperatures of $25^{\circ} \mathrm{C}$ in figure 5.9 a and $150^{\circ} \mathrm{C}$ in figure 5.9 b , interpolation of $Q_{R}$ is used.
The reverse recovery W.s/pulse, $J_{R}$, can be approximated by

$$
\begin{equation*}
J_{R}=V_{R} Q_{R} \tag{J}
\end{equation*}
$$

where $V_{R}$ is the reverse voltage applied to the diode just after turn-off. The reverse recovery average power loss is given by

$$
\begin{equation*}
P_{s}=V_{R} Q_{R} f_{s} \tag{W}
\end{equation*}
$$

The total average power loss is the algebraic sum of the steady-state conduction loss and the recovery loss.

$$
\begin{equation*}
W=\int v_{c e}(t) i_{c}(t) d t \tag{J}
\end{equation*}
$$

where the integration is performed over the switching transition period.


Figure 6.1. A typical IGBT transistor switching circuit incorporating a resistive load.
b-Explain in details a single phase bridge inverter?


FIGURE 14.8 Single-phase full-bridge VSI.

TABLE 14.2 Switch states for a full-bridge single-phase VSI

c-Define harmonic factor and total harmonic distortion?
Harmonic factor HF is the measure of the individual harmonics contribution+ the rms value of the fundamental component as rms values or the ratio of the rms values of the output voltage( individual harmonics+fundamental) $=\sum \mathrm{V}_{\mathrm{n}}$ to the rms value of the fundamental component $\mathrm{V}_{1}$
$H F=\sqrt{\frac{V_{n}^{2}-V_{1}^{2}}{V_{1}^{2}}}=$ harmonic $f, \quad H F_{n}=\frac{V_{n}}{V_{1}}=$ harmonic factor of the nth harmonic
Total harmonic distortion factor THD is the measure of the individual harmonics (closeness in shape between a waveform and its fundamental) only contribution as rms values or the ratio of the rms values of the output voltage harmonics ( individual harmonics ) $=\sum \mathrm{V}_{\mathrm{n}}$ to the rms value of the fundamental component $V_{1}$

$$
T H F=\frac{1}{V_{1}}\left(\sum_{n=2,3, . .}^{n=\alpha} V_{n}^{2}\right)^{\wedge} 0.5
$$

### 14.1.3v - Sinusoidal pulse-width modulation (pwm)

## 1-Natural sampling

## (a) Synchronous carrier

The output voltage waveform and method of generation for synchronous carrier, natural sampling sinusoidal pwm, suitable for the single-phase bridge of figure 14.1, are illustrated in figure 14.11. The switching points are determined by the intersection of the triangular carrier wave $f_{c}$ and the reference modulation sine


## (b) Asynchronous carrier

When the carrier is not an interger multiple of the modulation waveform, asynchronous modulation results. Because the output frequency, $f_{o}$, is usually variable over a wide range, it is difficult to ensure $f_{c}=n f_{o}$. To achieve synchronism, the carrier frequency must vary with frequency $f_{o}$. Simpler generating systems result if a fixed carrier frequency is used, resulting in asynchronism
2 - Regular sampling
Asynchronous carrier
When a fixed carrier frequency is used, usually no attempt is made to synchronise the modulation frequency. The output waveforms do not have quarter-wave symmetry which produces subharmonics. These subharmonics are insignificant if $f_{c} \gg f_{o}$, usually, $f_{c}>20 f_{o}$.

## Asymmetrical modulation

Asymmetrical modulation is produced when the carrier is compared with a stepped sine wave produced by sampling and holding at twice the carrier frequency, as shown in figure 14.13b. Each side of the triangular carrier about a sampling point intersects the stepped waveform at different step levels. The resultant pulse width is asymmetrical about the sampling point, as illustrated by the lower pulse in figure 14.14 for two modulation waveform magnitudes. The pulse width is given by

$$
\begin{equation*}
t_{p a}=\frac{1}{2 f_{c}}\left(1-1 / 2 M\left(\sin 2 \pi f_{o} t_{1}+\sin 2 \pi f_{o} t_{2}\right)\right) \tag{14.45}
\end{equation*}
$$




FIGURE 14.18 The space-vector representation.
a- Explain in details a dc-dc chopper?

A step-down dc chopper with a resistive load is shown in Fig. 13.1a. It is a series connection of a dc input voltage source $V_{S}$, controllable switch $S$, and load resistance $R$. In most cases, switch $S$ has unidirectional voltage-blocking capabilities and unidirectional current-conduction capabilities. Power electronic switches are usually implemented with power MOSFETs, IGBTs, MCTs, power BJTs, or GTOs. If an antiparallel diode is forms in a step-down chopper. The switch is being operated with a duty ratio $D$ defined as a ratio of the switch on time to the sum of the on and off times. For a constant frequency operation

$$
\begin{equation*}
D \equiv \frac{t_{\mathrm{on}}}{t_{\mathrm{on}}+t_{\mathrm{off}}}=\frac{t_{\mathrm{on}}}{T} \tag{13.1}
\end{equation*}
$$



FIGURE 13.1 DC chopper with resistive load: (a) circuit diagram; (b) output voltage waveform.
where $T=1 / f$ is the period of the switching frequency $f$. The average value of the output voltage is

$$
\begin{equation*}
V_{O}=D V_{S} \tag{13.2}
\end{equation*}
$$

b-A dc-dc chopper class A has a resistive load of $10 \Omega$ and the input dc voltage $\mathrm{Vs}=220 \mathrm{~V}$. The switch voltage drop is 2 V and the chopping frequency is 1 Khz , if the duty cycle is $50 \%$
Determine (i) the average output voltage (ii) the rms output voltage (iii) the output power?
$\mathrm{V}_{\mathrm{o}}=0.5 * 220=110 \mathrm{~V}, \mathrm{~V}_{\mathrm{rms}}=\left(0.5^{\wedge}{ }^{\wedge .5} * 220=155.6 \mathrm{~V}, \mathrm{Po}=0.5 * 220 * 220 * / 10=2420 \mathrm{~W}\right.$

## Question (3)

## [12] Points

a-Explain in details a single phase AC/AC voltage controller?

b-A single-phase full wave ac voltage controller has a resistive load of $10 \Omega$ and supply voltage of $120 \mathrm{~V}, 60 \mathrm{~Hz}$. The delay angles of the two thyristors are equal to $90^{\circ}$. (i) Draw the power circuit and its associated voltage and current waveforms. (ii) Determine the rms values of output voltage and current, (iii) the average value of the thyristor current?
$\left.\mathrm{V}_{\mathrm{o}}=120[(1 / \mathrm{pi})(\mathrm{pi}-90+(\sin 2 * 90) / 2))\right]^{\wedge} 0.5=84.85 \mathrm{~V}$,
$\mathrm{I}_{\mathrm{o}}=\mathrm{V}_{\mathrm{o}} / \mathrm{R}=84.85 / 10=8.485 \mathrm{~A}, \mathrm{I}_{\mathrm{th}}=\left[(1+\cos 90) 120^{*} 2^{\wedge 0.5}\right] / 2 \mathrm{pi}^{*} 10=2.7 \mathrm{~A}$

## P.T.O.

## Question (4)

[12] Points
a-Explain in details a single phase transformer tap-changer?

### 12.4 Single-phase transformer tap-changer

Figure 12.7 shows a single-phase tap changer where the tapped ac voltage supply can be provided by a tapped transformer or autotransformer.
Thyristor $T_{3}\left(T_{4}\right)$ is triggered at zero voltage cross-over, then under phase control $T_{1}$ $\left(\mathrm{T}_{2}\right)$ is turned on. The output voltage for a resistive load is defined by

$$
\begin{array}{cl}
v_{o}=\sqrt{2} V_{2} \sin \omega t & \text { (V) } \\
\text { for } 0 \leq \omega t \leq \alpha & \text { (rad) } \\
v_{o}=\sqrt{2} V_{1} \sin \omega t & \text { (V) }  \tag{12.23}\\
\text { for } \alpha \leq \omega t \leq \pi & \text { (rad) }
\end{array}
$$

where $\alpha$ is the phase delay angle and $v_{2}<\nu_{f}$.
For a resistive load the rms output voltage is

$$
\begin{equation*}
V_{\text {mour }}=\left[\frac{V_{2}^{2}}{\pi}(\alpha-1 / 2 \sin 2 \alpha)+\frac{V_{1}^{2}}{\pi}(\pi-\alpha+1 / 2 \sin 2 \alpha)\right]^{1 / 2} \tag{12.24}
\end{equation*}
$$


(a)

(b)

Figure 12.7. An ac voltage regulator using a tapped transformer:
(a) circuit connection and (b) output voltage waveform with a resistive load.

Initially $v_{2}$ is impressed across the load. Turning on $T_{1}\left(T_{2}\right)$ reverse-biases $T_{3}\left(T_{4}\right)$, hence $\mathrm{T}_{3}\left(\mathrm{~T}_{4}\right)$ turns off and the load voltage jumps to $v_{l}$. It is possible to vary the rms load voltage between $v_{2}$ and $v_{l}$. It is important that $\mathrm{T}_{1}\left(\mathrm{~T}_{2}\right)$ and $\mathrm{T}_{4}\left(\mathrm{~T}_{3}\right)$ do not conduct simultaneously, since such conduction short-circuits the transformer secondary.
With an inductive load circuit, when only $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$ conduct, the output current is

$$
\begin{equation*}
i_{o}=\frac{\sqrt{2} V}{Z} \sin (\omega t-\phi) \tag{12.25}
\end{equation*}
$$

where $Z=\sqrt{R^{2}+(\omega L)^{2}} \quad$ (ohms) $\quad \phi=\tan ^{-1} \omega L / R \quad$ (rad)
It is important that $\mathrm{T}_{3}$ and $\mathrm{T}_{4}$ are not fired until $\alpha \geq \phi$, when the load current must have reached zero. Otherwise a transformer secondary short circuit occurs through $T_{1}\left(T_{2}\right)$ and $\mathrm{T}_{4}\left(\mathrm{~T}_{3}\right)$.
For a resistive load, the thyristor rms currents for $\mathrm{T}_{3}, \mathrm{~T}_{4}$ and $\mathrm{T}_{1}, \mathrm{~T}_{2}$ respectively are

$$
\begin{align*}
& I_{\text {Tuxu }}=\frac{v_{z}}{2 R} \sqrt{\frac{1}{\pi}(2 \alpha-\sin 2 \alpha)} \\
& I_{\text {गmax }}=\frac{v_{1}}{2 R} \sqrt{\frac{1}{\pi}(\sin 2 \alpha-2 \alpha)+2 \pi} \tag{12.26}
\end{align*}
$$

The thyristor voltages ratings are both $v_{I}-v_{2}$, provided a thyristor is always conducting at any instant.
An extension of the basic operating principle is to use phase control on thyristors $T_{3}$ and $T_{4}$ as well as $T_{1}$ and $T_{2}$. It is also possible to use tap-changing in the primary circuit. The basic principle can also be extended from a single tap to a multi-tap transformer.
The basic operating principle of any multi-output tap changer, in order to avoid short circuits, independent of the load power factor is

- switch up in voltage when the load $V$ and $I$ have the same direction, delivering power
- switch down when $V$ and $I$ have the opposite direction, returning power.
b-A single phase transformer tap-changer has a primary voltage of $240 \mathrm{~V}, 60 \mathrm{~Hz}$. The secondary voltages are $\mathrm{V}_{1}=120 \mathrm{~V}$ and $\mathrm{V}_{2}=120 \mathrm{~V}$. If the load resistance is $10 \Omega$ and firing angles of thyristors T 1 and T 2 are $98^{\circ}$ and $\pi+98^{\circ}$ respectively. Determine (i) the load voltage, (ii) the rms current of the four thyristors (iii) the rms current of the secondary windings?

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{o}}=\left[\left(\mathrm{V}_{1}{ }^{2} / \mathrm{pi}\right)(\alpha-(\sin 2 \alpha) / 2]+\left(\left(\mathrm{V}_{1}+\mathrm{V}_{2}\right)^{2} / \mathrm{pi}\right)(\mathrm{pi}-\alpha+(\sin 2 \alpha) / 2)\right]^{0.5}=180 \mathrm{~V} \\
& \mathrm{I}_{\mathrm{R} 1(\mathrm{~T} 1, \mathrm{~T} 2)}=\left(\left(\mathrm{V}_{1}+\mathrm{V}_{2}\right) /(1.414 \mathrm{R})\right)[(1 / \mathrm{pi})(\mathrm{pi}-\alpha+(\sin 2 \alpha) / 2)]^{0.5}=10.9 \mathrm{~A} \\
& \left.\mathrm{I}_{\mathrm{R} 3(\mathrm{~T} 3, \mathrm{~T} 4)}=\left(\mathrm{V}_{1}\right) /(1.414 \mathrm{R})\right)[(1 / \mathrm{pi})(\alpha-(\sin 2 \alpha) / 2)]^{0.5}=6.5 \mathrm{~A} \\
& \mathrm{I}_{\mathrm{sec} .1(\mathrm{~T} 1, \mathrm{~T} 2)}=1.414 \mathrm{I}_{\mathrm{R} 1}=15.4 \mathrm{~A} \\
& \mathrm{I}_{\mathrm{sec} .2(\mathrm{~T} 1, \mathrm{~T} 2, \mathrm{~T}, \mathrm{~T} 4)}=\left[\left(1.414 \mathrm{I}_{\mathrm{R} 1}\right)^{2}+\left(1.414 \mathrm{I}_{\mathrm{R} 3}\right)^{2}\right]^{0.5}=17.94 \mathrm{~A}
\end{aligned}
$$

## Question (5)

## [12] Points

a- Explain in details a three phase bridge inverter?

igure 14.7. A three-phase bridge inverter employing $120^{\circ}$ switch conduction th a resistive star load: (a) the bridge circuit showing $T_{1}$ and $T_{2}$ conducting and (b circuit voltage and current waveforms.


FIGURE 14.13 Three-phase VSI topology.

c- Design a three-phase PWM inverter that drives a 30hp, variable speed, 3-phase, 12-pole, $460 \mathrm{~V}, 60 \mathrm{~Hz}$ induction motor. The DC supply is 300 V and $\mathrm{m}_{\mathrm{a}}=0.8$ and $\mathrm{m}_{\mathrm{f}}=15$. Draw a block diagram of the system and use the following table to calculate the fundamental frequency line to line voltage and some dominant harmonics of line to line voltages.

|  | $\mathrm{m}_{\mathrm{a}}$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| h | 0.2 | 0.4 | 0.6 | 0.8 | 1 |  |
| 1 | 0.122 | 0.245 | 0.367 | 0.490 | 0.612 |  |
| $\mathrm{~m}_{\mathrm{f}} \pm 2$ | 0.010 | 0.037 | 0.080 | 0.135 | 0.195 |  |


| $2 \mathrm{~m}_{\mathrm{f}} \pm 1$ | 0.116 | 0.20 | 0.227 | 0.192 | 0.111 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $3 \mathrm{~m}_{\mathrm{f}} \pm 2$ | 0.027 | 0.085 | 0.124 | 0.108 | 0.038 |

$$
\begin{aligned}
& \mathrm{V}_{1}=0.612 * \mathrm{~m}_{\mathrm{a}} * \mathrm{~V}_{\mathrm{s}}=0.612 * 300 * 0.49=90 \mathrm{~V} \\
& \mathrm{~V}_{13,17}=0.612 * \mathrm{~m}_{\mathrm{a}} * \mathrm{~V}_{\mathrm{s}}=0.612 * 300 * 0.135=32.4 \mathrm{~V} \\
& \mathrm{~V}_{29,31}=0.612 * \mathrm{~m}_{\mathrm{a}} * \mathrm{~V}_{\mathrm{s}}=0.612 * 300 * 0.192=46.1 \mathrm{~V} \\
& \mathrm{~V}_{43,47}=0.612 * \mathrm{~m}_{\mathrm{a}} * \mathrm{~V}_{\mathrm{s}}=0.612 * 300 * 0.108=25.9 \mathrm{~V}
\end{aligned}
$$

