Benha University
College of Engineering at Benha
Department of Mechanical Eng.

Subject: **Fluid Mechanics**

Model Answer of the Final Corrective Exam

Date of the Exam Dec/2٨/201٤

نموذج الاجابة لمادة : مادة ميكانيا الموائع م1111

تاريخ الامتحان الاثنين 29 ديسمبر ٢٠١٤

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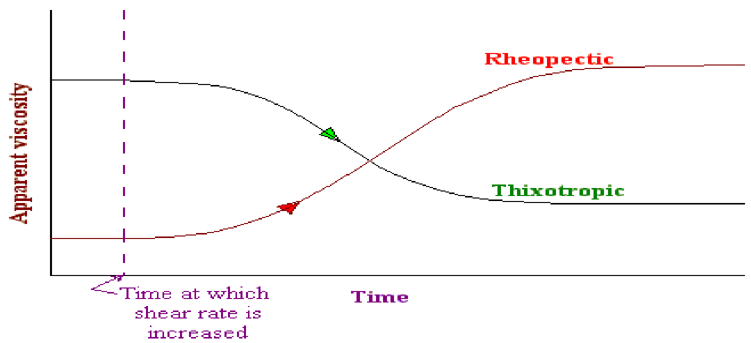
1-a) i) ***Thixotropic*** *fluids*: for which the dynamic viscosity decreases with the time for which shearing forces are applied. e.g. thixotropic jelly paints.

Rheopectic *fluids*: Dynamic viscosity increases with the time for which shearing forces are applied. e.g. gypsum suspension in water.



Effect of sudden change of shear rate on apparent viscosity of time-dependent fluids

Figure 1

ii) **Surface tension** is a phenomena that makes the interface between two fluids or between liquid and water behaves as elastic membrane.

iii) **Streakline** is a line passing through fluid particles originated from a given point (have earlier passed through a certain point in the flow field, **Timelines** are lines connecting specified particles at different time instants.

iv) ***Visco-elastic fluids*****:** Some fluids have elastic properties, which allow them to spring back when a shear force is released. e.g. egg white.

v) P**otential flow:** Flow whose vorticity equal zero or Curl **V** = 0

vi) where the vector operation ($\nabla \times \mathbf{V}$) is referred to as the curl of the velocity vector **V**. The vorticity vector, ζ, is defined as twice the rotation vector:

$$\zeta = 2\boldsymbol{\omega} = \nabla \times \mathbf{V}$$

vii) Circulation is defined as the line integral around the curve of the arc length **ds** times the tangential component of velocity. Shear stress for the element is thus given by

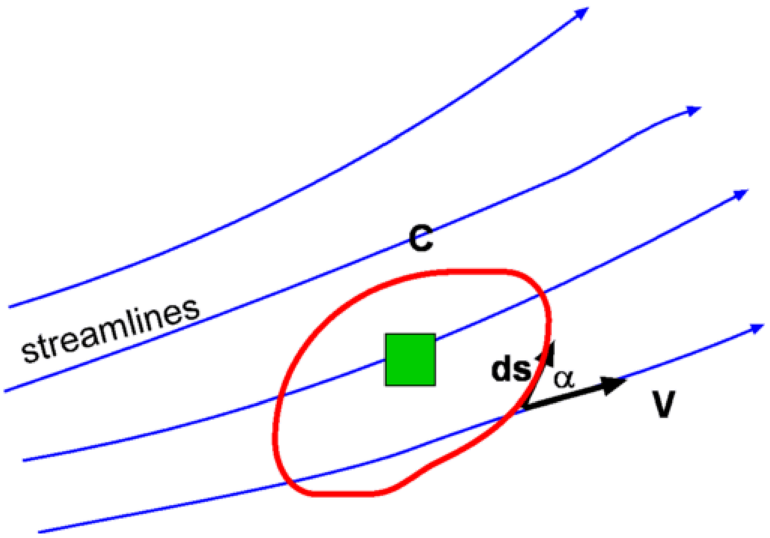


Figure 2: Definition of Circulation

1-b) The relationship between shear stress and the velocity gradient (rate of shear strain) in the fluid. These relationships can be seen in the graph below for several categories

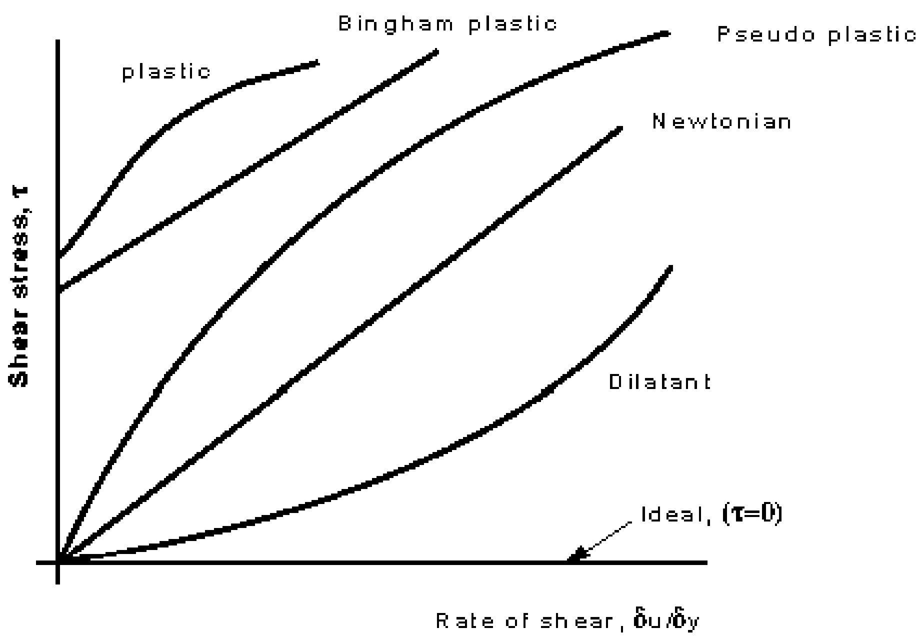


Figure 3 Shear stress vs. Rate of shear strain δu/δy

Newtonian fluids:

Fluids which obey the Newton's law of viscosity are called as Newtonian fluids. Newton's law of viscosity is given by

$\tau = \mu\, du/dy$, where τ = shear stress and $\boldsymbol{\mu}$ = viscosity of fluid

du/dy = shear rate, rate of strain or velocity gradient

All gases and most liquids which have simpler molecular formula and low molecular weight such as water, benzene, ethyl alcohol, CCl_4, hexane and most solutions of simple molecules are Newtonian fluids.

Properties are independent of time under shear.

Bingham-plastic: Resist a small shear stress but flow easily under larger shear stresses. e.g. tooth-paste, jellies, and some slurries.

Pseudo-plastic:

Most non-Newtonian fluids fall into this group . No minimum shear stress necessary and the viscosity decreases with rate of shear, e.g. polymer solutions, blood, colloidal substances like clay, milk and cement.. Pseudo plastic fluids are also called as Shear thinning fluids. At low shear rates (du/dy) the shear thinning fluid is more viscous than the Newtonian fluid, and at high shear rates it is less viscous.

○ ***Dilatant fluids***: Viscosity increases with increasing velocity gradient. They are uncommon, but suspensions of starch and sand behave in this way. Dilatant fluids are also called as shear thickening fluids. *;* Viscosity increases with rate of shear e.g. quicksand.

1-c) $\mu = 7.2 \times 10^{-1} = 0.72 kg/m.s$

$$\tau = \mu \frac{du}{dy} \rightarrow du = rw = \frac{r(2\pi n)}{60} = \frac{0.18x2\pi x200}{60} = 3.77m$$

$$\tau = \mu \frac{du}{dy} \rightarrow \tau = 0.72 \times \frac{3.77}{1.5 \times 10^{-4}} = 18096 Pa$$

$$F = \tau A = \tau \pi DL \rightarrow F = 20466 N$$

$$Power = Fr\omega = 77155 W$$

2-a) The compressibility of the fluid is given by: $k = \frac{1}{\rho}\frac{d\rho}{dp}$

For isentropic process $\frac{p}{\rho^{\gamma}} = C \rightarrow \frac{dp}{d\rho} = C\gamma\rho^{\gamma-1} = \frac{p}{\rho^{\gamma}}\gamma\rho^{\gamma-1} \rightarrow \frac{dp}{d\rho} = \frac{\gamma p}{\rho}$

$\therefore$ the isentropic compressibility $k_s = \frac{1}{\rho}\frac{\rho}{\gamma p} = \frac{1}{\gamma p}$

2-b) **Solution:** The total beam volume is $3(.1)^2 = 0.03\ m^3$, and therefore its weight is W = (0.65)(9790)(0.03) = 190.9 N, acting at the centroid, 1.5 m down from point A. Meanwhile, if the submerged length is H, the buoyancy is B = $(9790)(0.1)^2H$ = 97.9H Newton, acting at H/2 from the lower end. Sum moments about point A:

$$\sum M_A = 0 = (97.9H)(3.0 - H/2)\cos\theta - 190.9(1.5\cos\theta),$$

$$\text{or:} \quad H(3 - H/2) = 2.925, \quad \text{solve for} \quad H \approx 1.225 \text{ m}$$

Geometry: 3 – H = 1.775 m is out of the water, or: $\sin\theta$= 1.0/1.775, or $\theta \approx$ **34.3°**

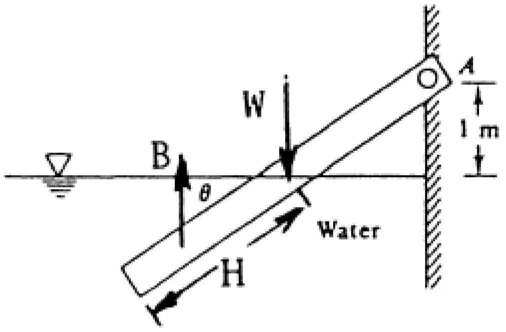


FIGURE 4

2-c

Solution: Let H = (h – 1 meter) be the depth down to the level AB. The forces on AB and BC are shown in the freebody at right. The moments of these forces about B are equal when the gate opens:

$$\sum M_B = 0 = \gamma H(0.2)b(0.1) = \gamma\left(\frac{H}{2}\right)(Hb)\left(\frac{H}{3}\right)$$

$$\text{or:} \quad H = 0.346 \text{ m}, \quad h = H + 1 = \mathbf{1.346\ m} \quad \textit{Ans.}$$

This solution is independent of both the water density and the gate width b into the paper.

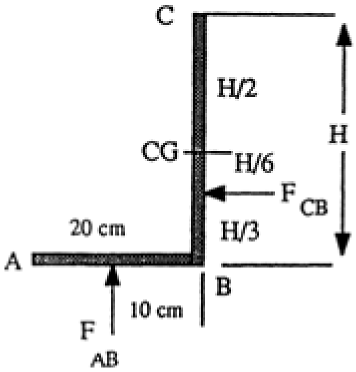


Figure 5

3-a) Solution

Part i) The cup contains 7 cm of coffee. The remaining distance of 3 cm up to the lip must equal the distance $h/2$ in next figure.. Thus

$$\frac{h}{2} = 0.03 \text{ m} = \frac{\Omega^2 R^2}{4g} = \frac{\Omega^2(0.03 \text{ m})^2}{4(9.81 \text{ m/s}^2)}$$

Solving we obtain

$$\Omega^2 = 1308 \qquad \text{or} \qquad \Omega = 36.2 \text{ rad/s} = 345 \text{ r/min}$$

To compute the pressure, it is convenient to put the origin of coordinates r and z at the bottom of the free-surface depression, as shown in Fig.. The gage pressure here is p_o=0, and point A is at (r, z)= (3 cm, _4 cm). We can then evaluate

$$p_A = 0 - (1010 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(-0.04 \text{ m})$$
$$+ \tfrac{1}{2}(1010 \text{ kg/m}^3)(0.03 \text{ m})^2(1308 \text{ rad}^2/\text{s}^2)$$
$$= 396 \text{ N/m}^2 + 594 \text{ N/m}^2 = 990 \text{ Pa}$$

This is about 43 percent greater than the still-water pressure $p_A = 694$ Pa.

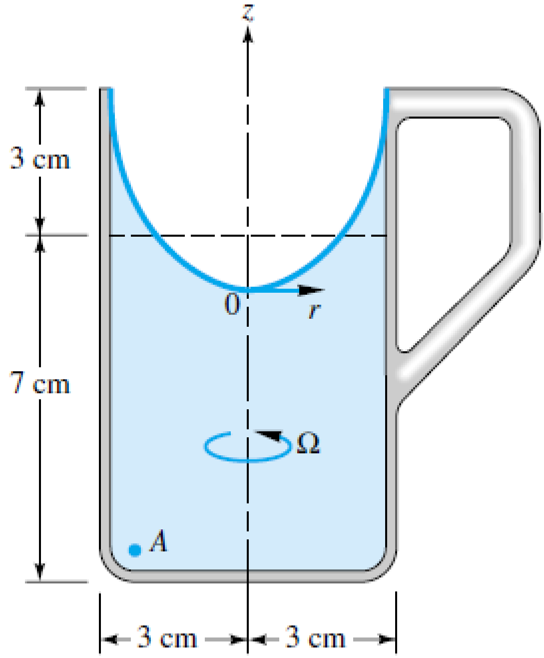


Figure 6

3-b)Center of Bouncy is the point of action of the up thrust acting on the floating or immersed bodies..
A vertical line drawn upward from B_ intersects the line of symmetry at a point *M*, called the *metacenter*.

Stability of floating bodies

There are two cases

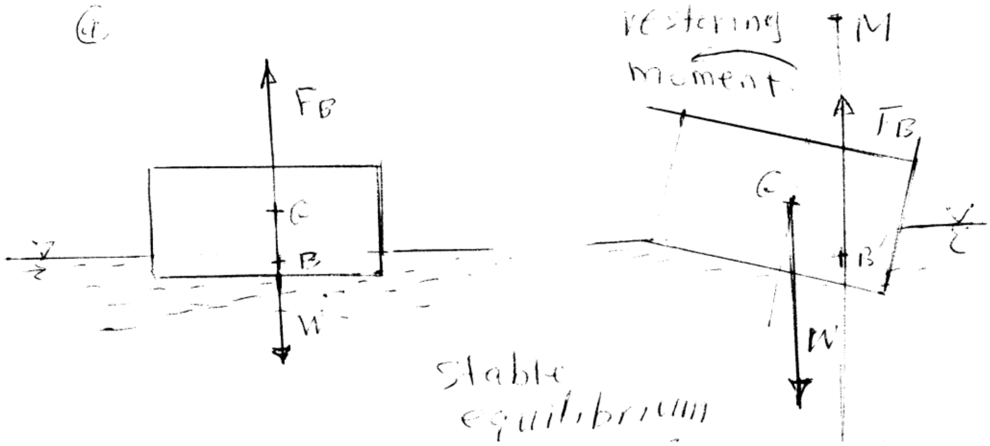


Figure 7-a

Case 1: Stable equilibrium , when the floating body is tilted to the right, a couple is created due to the deviation between the lines of actions of weight and buoyant force. This moment restore the body to its original position when the metacenter, M, is above the center of gravity, G.(figure 7-a)

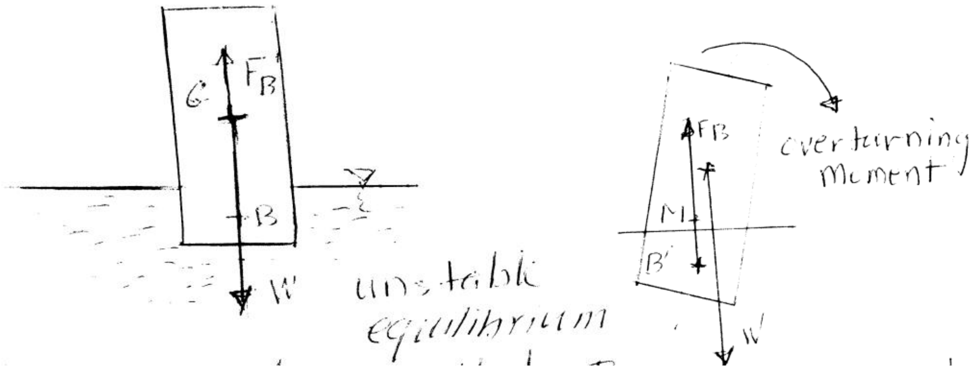


Figure 7-b

Case 2: Unstable equilibrium, when the floating body is tilted to the right, a couple is created due to the deviation between the lines of actions of weight and buoyant force. This moment turn the body over to when the metacenter, M, is below the center of gravity, Figure 7-b

3-c)

c)) i-

$$\vec{V} = (x^2 - y^2 + x)i - (2xy + y)j$$

$$\vec{V} = ui + vj \rightarrow\rightarrow u = x^2 - y^2 + x,\ v = -(2xy + y)$$

$$\text{the acceleration } \vec{a} = a_x i + a_y j$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \rightarrow\rightarrow a_x = (x^2 - y^2 + x)(2x+1) + -(2xy + y)(\text{-}2y) = 2x^3 + 2xy^2 + x + 3x^2 + y^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \rightarrow\rightarrow a_y = (x^2 - y^2 + x)(\text{-}2y) + -(2xy + y)(\text{-}2x\text{-}1)) = 2y^3 + 2yx^2 + 2xy + y$$

$$a_x = 18, a_y = 26 \rightarrow \vec{a} = 18i + 26j$$

ii) The flow will represent a physical flow if $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, for this flow we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2x + 1 - (2x+1) = 0$$

$\therefore$ The flow is a physical flow.

$$\text{iii)}\ u = x^2 - y^2 + x = \frac{\partial \psi}{\partial y} \rightarrow \psi = x^2 y - \frac{y^3}{3} + xy + f(x)$$

$$v = -2xy - y = -\frac{\partial \psi}{\partial x} \rightarrow \psi = x^2 y + xy + g(y)$$

$$\therefore \psi = x^2 y - \frac{y^3}{3} + xy$$

$$u = x^2 - y^2 + x = \frac{\partial \phi}{\partial x} \rightarrow \phi = -y^2 x + \frac{x^3}{3} + \frac{x^2}{2} + g(y)$$

$$v = -2xy - y = \frac{\partial \phi}{\partial y} \rightarrow \phi = -xy^2 - \frac{y^2}{2} + f(x)$$

$$\phi = -y^2 x + \frac{x^3}{3} + \frac{x^2}{2} + - \frac{y^2}{2}$$

iv) The vorticity is given by : $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -2y + 2y = 0$

4-a)i)

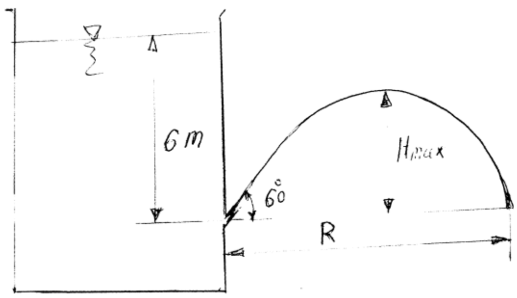


Figure 7

The maximum height H_{max} is given from the relation;

$H_{max} = H \sin^2\theta = 6.0x0.25 = 1.5$ m;

The range R is given by the equation: $R = 2H\sin 2\theta \rightarrow R = 2 \times 6 \times \sin 60^{\circ}$

$$R = 6\sqrt{3} = 10.392\text{m}$$

ii) The time to empty the tank is calculated as follows:

$$-A_T \frac{dh}{dt} = C_d A_o \sqrt{2gh} \rightarrow dt = -\frac{A_T}{C_d A_o \sqrt{2g}} \frac{dh}{\sqrt{h}}$$

$$t = -\frac{A_T}{C_d A_o \sqrt{2g}} \int_6^0 \frac{dh}{\sqrt{h}} = -\frac{2A_T \sqrt{h}}{C_d A_o \sqrt{2g}} \updownarrow_6^0 \rightarrow t = 1382.5\text{sec}$$

4-b)

Solution: Let "1" be at the reservoir surface and "2" be at the nozzle exit, as shown. We need to know the exit velocity:

As shown in Figure 8

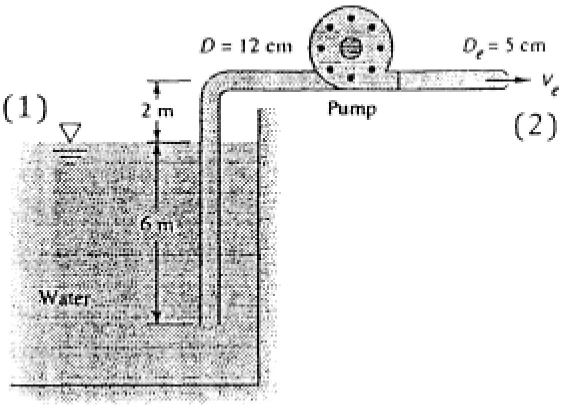


Figure 8

$$V_2 = Q/A_2 = \frac{220/3600}{\pi(0.025)^2} = 31.12\ \frac{m}{s},\quad \text{while } V_1 \approx 0 \text{ (reservoir surface)}$$

Now apply the steady flow energy equation from (1) to (2):

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f - h_p,$$

$$\text{or:}\quad 0+0+0 = 0+(31.12)^2/[2(9.81)]+2+5-h_p,\quad \text{solve for } h_p \approx 56.4 \text{ m.}$$

$$\text{The pump power}\quad P = \rho g Q h_p = (998)(9.81)(220/3600)(56.4)$$
$$= 33700 \text{ W} = \mathbf{33.7\ kW}\quad \textit{Ans.}$$

5-a)

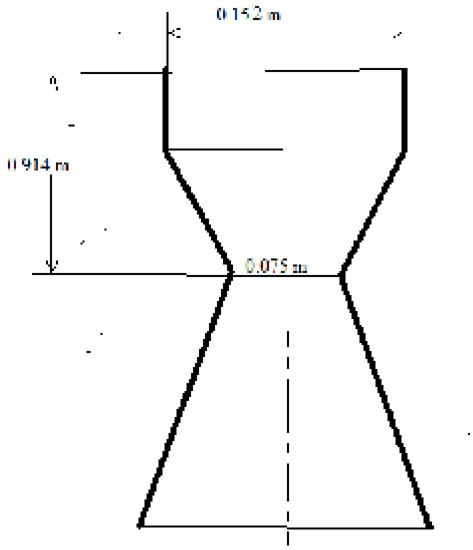


Figure 9

From the question:

$d_1 = 0.152m \qquad A_1 = 0.01814m$

$d_2 = 0.076m \qquad A_2 = 0.00454m$

$\rho = 800 kg/m^3$

$C_d = 0.97$

Apply Bernoulli:

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

1. $p_1 = p_2$

$$\frac{u_1^2}{2g} + z_1 = \frac{u_2^2}{2g} + z_2$$

By continuity:

$$Q = u_1 A_1 = u_2 A_2$$

$$u_2 = u_1 \frac{A_1}{A_2} = u_1 4$$

$$\frac{u_1^2}{2g} + 0.914 = \frac{16u_1^2}{2g}$$

$$u_1 = \sqrt{\frac{0.914 \times 2 \times 9.81}{15}} = 1.0934 \; m/s$$

$$Q = C_d A_1 u_1$$

$$Q = 0.96 \times 0.01814 \times 1.0934 = 0.019 m^3/s$$

ii)

$$p_1 - p_2 = 15170$$

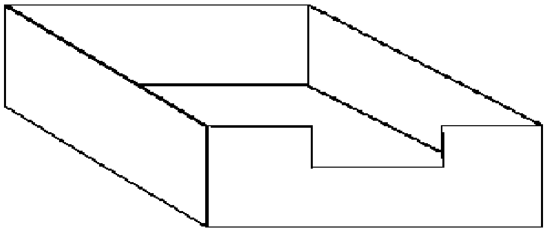
$$\frac{p_1 - p_2}{\rho g} = \frac{u_2^2 - u_1^2}{2g} - 0.914$$

$$\frac{15170}{\rho g} = \frac{Q^2\left(220.43^2 - 55.11^2\right)}{2g} - 0.914$$

$$55.8577 = Q^2\left(220.43^2 - 55.11^2\right)$$

$$Q = 0.035 m^3/s$$

5-b)



From the question $A = 60\,000\ m^2$, $Q = 0.678\ h^{3/2}$

Write the equation for the discharge in terms of the surface height change:

$$Q\,\delta t = -A\,\delta h$$

$$\delta t = -\frac{A}{Q}\delta h$$

Integrating between h_1 and h_2, to give the time to change surface level

$$T = -\int_{h_1}^{h_2} \frac{A}{Q}\,dh$$

$$= -\frac{60000}{0.678}\int_{h_1}^{h_2} \frac{1}{h^{3/2}}\,dh$$

$$= 2 \times 88495.58\left[h^{-1/2}\right]_{h_1}^{h_2}$$

From the question $T = 3600\ sec$ and $h_1 = 0.6m$

$$3600 = 176991.15\left[h_2^{-1/2} - 0.6^{-1/2}\right]$$

$$h_2 = 0.5815m$$

Total depth = 3.4 + 0.58 = 3.98m