Benha University | **Jan. 2015**
Benha Faculty of Engineering | **Electrical Department**
Electrical Engineering and Circuit Analysis(a) (E1101) | **1st Year Electrical**
Dr.Wael Abdel-Rahman Mohamed | **Time: 3 Hrs**

Exam with model answer

<u>Question (1):</u> [8 Marks]

Find the equivalent resistance R_{ab} in the circuit in Fig.1

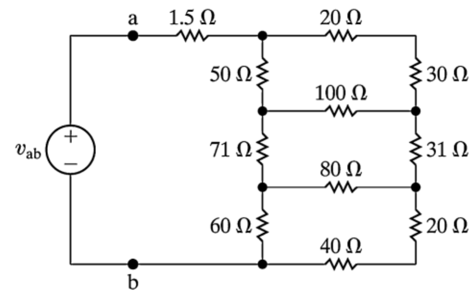


Fig.1

Convert the upper delta to a wye.

$$R_1 = \frac{(50)(50)}{200} = 12.5\,\Omega$$

$$R_2 = \frac{(50)(100)}{200} = 25\,\Omega$$

$$R_3 = \frac{(100)(50)}{200} = 25\,\Omega$$

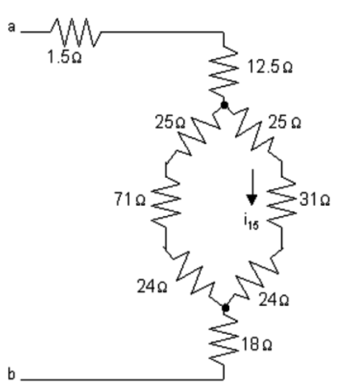
Convert the lower delta to a wye.

$$R_4 = \frac{(60)(80)}{200} = 24\,\Omega$$

$$R_5 = \frac{(60)(60)}{200} = 18\,\Omega$$

$$R_6 = \frac{(80)(60)}{200} = 24\,\Omega$$

Now redraw the circuit using the wye equivalents.



$$R_{ab} = 1.5 + 12.5 + \frac{(120)(80)}{200} + 18 = 14 + 48 + 18 = 80\,\Omega$$

Question (2): **[12 Marks]**

Use the node voltage method to find the power developed by the 20 V source in the circuit in Fig.2

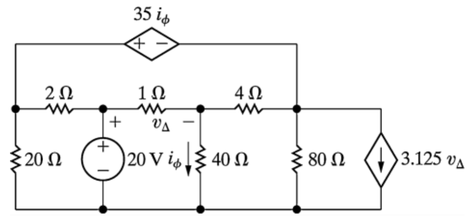
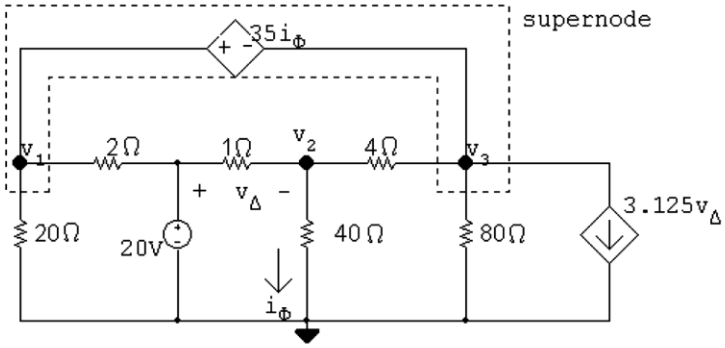


Fig.2



Node equations:

$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_3 - v_2}{4} + \frac{v_3}{80} + 3.125v_\Delta = 0$$

$$\frac{v_2}{40} + \frac{v_2 - v_3}{4} + \frac{v_2 - 20}{1} = 0$$

Constraint equations:

$$v_\Delta = 20 - v_2$$

$$v_1 - 35i_\phi = v_3$$

$$i_\phi = v_2/40$$

Solving, $v_1 = -20.25$ V; $\quad v_2 = 10$ V; $\quad v_3 = -29$ V

Let i_g be the current delivered by the 20 V source, then

$$i_g = \frac{20 - (20.25)}{2} + \frac{20 - 10}{1} = 30.125 \text{ A}$$

$$p_g \text{ (delivered) } = 20(30.125) = 602.5 \text{ W}$$

<u>Question (3):</u> [8 Marks]

Use the mesh-current method to find the total power developed in the circuit in Fig.3

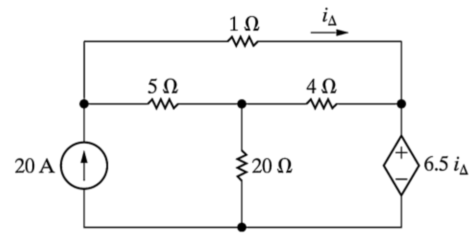
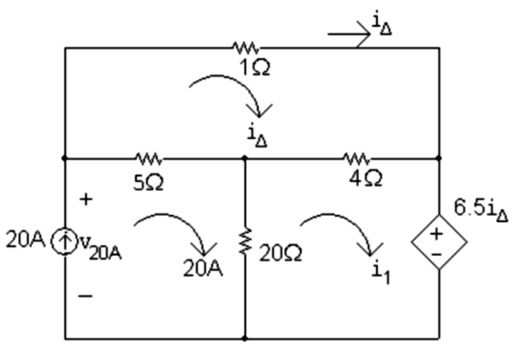


Fig.3



Mesh equations:

$$10i_\Delta - 4i_1 = 0$$

$$-4i_\Delta + 24i_1 + 6.5i_\Delta = 400$$

Solving, $i_1 = 15$ A; $\quad i_\Delta = 16$ A

$$v_{20\text{A}} = 1i_\Delta + 6.5i_\Delta = 7.5(16) = 120 \text{ V}$$

$$p_{20\text{A}} = -20v_{20\text{A}} = -(20)(120) = -2400 \text{ W (del)}$$

$$p_{6.5i_\Delta} = 6.5i_\Delta i_1 = (6.5)(16)(15) = 1560 \text{ W (abs)}$$

Therefore, the independent source is developing 2400 W, all other elements are absorbing power, and the total power developed is thus 2400 W.

CHECK:

$$p_{1\Omega} = (16)^2(1) = 256 \text{ W}$$

$$p_{5\Omega} = (20 - 16)^2(5) = 80 \text{ W}$$

$$p_{4\Omega} = (1)^2(4) = 4 \text{ W}$$

$$p_{20\Omega} = (20 - 15)^2(20) = 500 \text{ W}$$

$$\sum p_{\text{abs}} = 1560 + 256 + 80 + 4 + 500 = 2400 \text{ W (CHECKS)}$$

<u>Question (4):</u> [12 Marks]

The variable dc current source in the circuit in Fig.4 is adjusted so that the power developed by the 4 A current source is zero. Find the value of $\boldsymbol{i_{dc}}$.

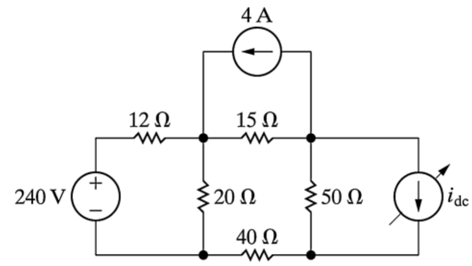
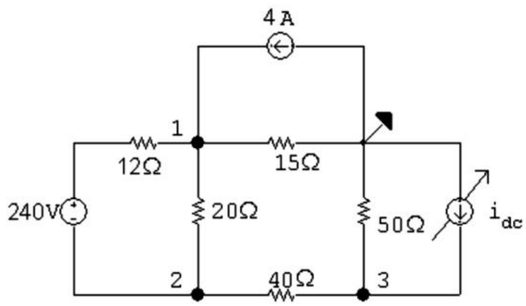


Fig.4

Choose the reference node so that a node voltage is identical to the voltage across the 4 A source; thus:



Since the 4 A source is developing 0 W, v_1 must be 0 V.

Since v_1 is known, we can sum the currents away from node 1 to find v_2; thus:

$$\frac{0-(240+v_2)}{12}+\frac{0-v_2}{20}+\frac{0}{15}-4=0$$

$$\therefore \quad v_2=-180 \text{ V}$$

Now that we know v_2 we sum the currents away from node 2 to find v_3; thus:

$$\frac{v_2+240-0}{12}+\frac{v_2-0}{20}+\frac{v_2-v_3}{40}=0$$

$$\therefore \quad v_3=-340 \text{ V}$$

Now that we know v_3 we sum the currents away from node 3 to find i_{dc}; thus:

$$\frac{v_3}{50}+\frac{v_3-v_2}{40}=i_{\text{dc}}$$

$$\therefore \quad i_{\text{dc}}=-10.8 \text{ A}$$

Question (5): [12 Marks]

The variable resistor R_o in the circuit in Fig.5 is adjusted until the power dissipated in the resistor is 250 W. Use **Thévenin's theorem** to find the values of R_o that satisfy this condition.

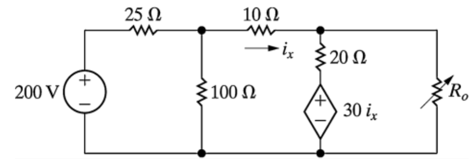
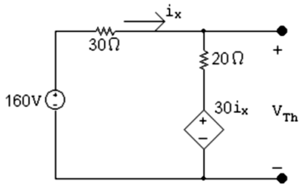


Fig.5

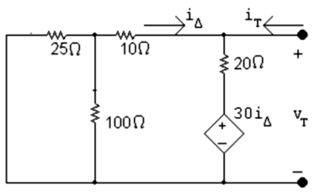
We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2 \text{ A}$$

$$V_{\rm Th} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100 \text{ V}$$

Using the test-source method to find the Thévenin resistance gives

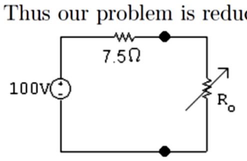


$$i_{\rm T} = \frac{v_{\rm T}}{30} + \frac{v_{\rm T} - 30(-v_{\rm T}/30)}{20}$$

$$\frac{i_{\rm T}}{v_{\rm T}} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{\rm Th} = \frac{v_{\rm T}}{i_{\rm T}} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$

$$\frac{10^4}{R_o^2 + 15R_o + 56.25} R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

$$R_o^2 - 25R_o + 56.25 = 0$$

$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R_o = 22.5\,\Omega$$

$$R_o = 2.5\,\Omega$$

<u>Question (6):</u> [8 Marks]

The op amps in the circuit in Fig.6 are ideal.

a) Find i_a.

b) Find the value of the left and right voltage sources for which $i_a = i_{a\,max}$

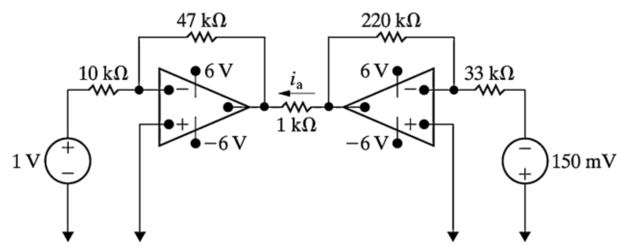


Fig.6

[a] Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-47}{10}(1) = -4.7 \text{ V}; \qquad v_{o2} = \frac{-220}{33}(-0.15) = 1.0 \text{ V}$$

$$i_a = \frac{v_{o2} - v_{o1}}{1000} = 5.7\,\text{mA}$$

[b] for $i_{a\,max}$, the output of each amplifier is the saturation but one is +ve and the other is –ve

The output of the right amplifier is +Vcc = +6V

The output of the left amplifier is –Vcc = -6V

$i_{a\,max}$ = (6 – (-6)) / 1kΩ = 12 mA

With best wishes