

Answers of Final Written Exam

Question ①

﴿26 marks﴾

- (a) [12 marks] A string is vibrating sinusoidally with amplitude 10 cm and a frequency of 50.0 Hz producing a wave in the negative x -direction and making four complete cycles in a distance 120 cm. The total length of the string is 2 m, and it has a mass of 180 g. **(i)** Write the function that describes this wave knowing that the string deviation is zero at $x = 0$ and $t = 0$. **(ii)** Determine the tension in the string. **(iii)** Determine the power being supplied to the string.

Answer:

$A = 10 \text{ cm}$, $f = 50 \text{ Hz}$, the direction of propagation is the negative x -direction,

$l = 2 \text{ m}$, $m = 180 \text{ g}$.

$$\lambda = \frac{120}{4} = 30 \text{ cm} = 0.3 \text{ m} \quad \rightarrow \quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.3} = \frac{20\pi}{3} \text{ rad/m}$$

$$\omega = 2\pi f = 2\pi(50) = 100\pi \text{ rad/s}$$

$$\text{i. } y = A \sin(kx + \omega t + \varphi) = 0.10 \sin\left(\frac{20\pi}{3}x + 100\pi t\right)$$

$$\text{ii. } v = \sqrt{\frac{F}{\mu}} \quad \rightarrow \quad F = \mu v^2 = \frac{0.18}{2} ((0.3)(50))^2 = 20.25 \text{ N}$$

$$\text{iii. } P = \frac{1}{2} \mu v A^2 \omega^2 = \frac{1}{2} (0.09)(15)(0.1)^2 (100\pi)^2 = 666.2 \text{ W}$$

- (b) [4 marks] A vacuum cleaner produces a sound wave in air with a measured sound level of 70.0 dB. **(i)** Calculate the intensity of this sound wave at the measured point, **(ii)** Find the pressure amplitude of this sound wave at the same point.

Answer:

$$\text{i. } \beta = 10 \log\left(\frac{I}{I_0}\right) = 70 \quad \rightarrow \quad I = 10^7 (1 \times 10^{-12}) I_0 = 10^{-5} \text{ W/m}^2$$

$$\text{ii. } I = \frac{1}{2} \rho v s_{\max}^2 \omega^2, \quad \Delta P_{\max} = \rho v s_{\max} \omega \quad \rightarrow \quad \frac{(\Delta P_{\max})^2}{I} = 2\rho v$$

$$\Delta P_{\max} = \sqrt{2I\rho v} = 92.2 \times 10^{-3} \text{ Pa}$$

- (c) [6 marks] Consider an observer moving with constant velocity 170 m/s toward a fixed sound source. When the observer was at a distance 10 m from the source, he heard a frequency of 300 Hz, and the sound level is found to be 25 dB at that point. **(i)** Find the natural frequency of the source. **(ii)** When the observer becomes at a distance 1 m from the source moving with the same velocity, find the frequency heard by the observer, and the sound level at this distance.

Answer:

i. $f' = \left(\frac{v + v_o}{v}\right)f \rightarrow f' = \left(\frac{v}{v + v_o}\right)f' = \left(\frac{340}{340 + 170}\right)(300) = 200 \text{ Hz}$

ii. **The frequency will be the same because the velocity is the same, so,
 $f = 300 \text{ Hz}$**

$$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1}\right) = 20 \log \left(\frac{r_1}{r_2}\right) = 20 \log \left(\frac{10}{1}\right) = 20$$

$$\beta_2 = \beta_1 + 20 = 45 \text{ dB}$$

(d) [4 marks] The nearest antinode to the closed end of a closed-end pipe was found to be at 10 cm from the closed end. What is distance between the following anti-node and the closed end of the pipe?

Answer:

$$L_1 = \frac{\lambda}{4} = 10 \text{ cm} \quad \rightarrow$$

$$\lambda = 4L_1 = 40 \text{ cm}$$

$$L_2 = \frac{3\lambda}{4} = \frac{3(40)}{4} = 30 \text{ cm}$$

Question ②

《24 marks》

(a) [6 marks] In a double-slit experiment, the distance between slits is 5.0 mm and the slits are 1.0 m from the screen. Two interference patterns can be seen on the screen: one due to light of wavelength 480 nm, and the other due to light of wavelength 600 nm. What is the separation on the screen between the third-order ($m = 3$) bright fringes of the two interference patterns?

Answer:

$$\begin{aligned} \Delta y &= \left[\frac{mD}{d}\right](\lambda_2 - \lambda_1) \\ &= \left[\frac{3(1.0 \text{ m})}{5.0 \times 10^{-3} \text{ m}}\right](600 \times 10^{-9} \text{ m} - 480 \times 10^{-9} \text{ m}) \\ \Delta y &= 7.2 \times 10^{-5} \text{ m} \end{aligned}$$

(b) [6 marks] A thin film of acetone ($n = 1.25$) coats a thick glass plate ($n = 1.50$). White light in air is incident normal to the film. In the reflections, fully destructive interference occurs at a wavelength 600 nm and fully constructive interference at a wavelength 700 nm. Calculate the thickness of the acetone film

Answer:

Destructive interference

$$2nt = \left(m + \frac{1}{2}\right) \lambda \quad (\lambda = 600 \text{ nm})$$

$$t = \left(m + \frac{1}{2}\right) \frac{\lambda}{2n} \quad (m = 0, 1, 2, 3, \dots)$$

$$t = 120, 360, 600, 840, \dots \text{ nm}$$

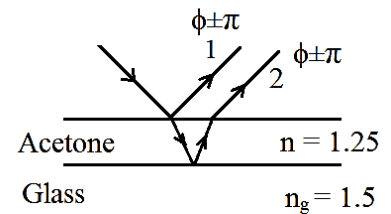
Constructive interference

$$2nt = m \lambda \quad (\lambda = 700 \text{ nm})$$

$$t = \frac{m \lambda}{2n} \quad (m = 1, 2, 3, 4, \dots)$$

$$t = 280, 560, 840, 1120, \dots \text{ nm}$$

The lowest number these lists have in common is $t = 840 \text{ nm}$



- (c) [6 marks] A slit 1.00 mm wide is illuminated by light of wavelength 589 nm. We see a diffraction pattern on a screen 3.00 m away. What is the distance between the first two diffraction minima on the same side of the central diffraction maximum?

Answer:

$$\Delta y = \left[\frac{\Delta m D \lambda}{a} \right]$$

$$= \left[\frac{(2-1)(3.0 \text{ m})(589 \times 10^{-9} \text{ m})}{1.0 \times 10^{-3} \text{ m}} \right]$$

$$\Delta y = 1.77 \times 10^{-3} \text{ m}$$

- (d) [6 marks] A diffraction grating is made up of slits of width 300 nm with separation 900 nm. The grating is illuminated by monochromatic plane waves of wavelength $\lambda = 600 \text{ nm}$ at normal incidence. How many maxima are there in the full diffraction pattern?

Answer:

$$\therefore d \sin(\theta) = m \lambda$$

$$\therefore \sin(\theta) \leq 1$$

$$\therefore \frac{m \lambda}{d} \leq 1$$

$$\therefore m \leq \frac{d}{\lambda} = \left[\frac{900}{600} \right] = 1.5$$

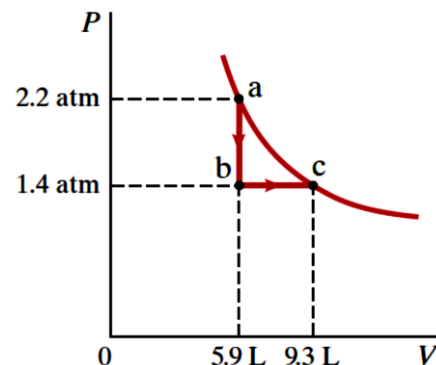
$$\therefore m = 1$$

So the number of maxima are $(2m+1) = 3$

Question ③

22 marks

- (a) [10 marks] Consider the following two-step process. Heat is allowed to flow out of an ideal gas at constant volume so that its pressure drops from 2.2 atm to 1.4 atm. Then the gas expands at constant pressure, from a volume of 5.9 L to 9.3 L, where the temperature reaches its original value. Calculate (i) the total work done by the gas in the process, (ii) the change in internal energy of the gas in the process, and (iii) the total heat flow into or out of the gas.



Answer:

- (a) No work is done during the first step, since the volume is constant. The work in the second step is given by $W = P\Delta V$.

$$W = P\Delta V = (1.4 \text{ atm}) \left(\frac{1.01 \times 10^5 \text{ Pa}}{1 \text{ atm}} \right) (9.3 \text{ L} - 5.9 \text{ L}) \left(\frac{1 \times 10^{-3} \text{ m}^3}{1 \text{ L}} \right) = \boxed{480 \text{ J}}$$

- (b) Since there is no overall change in temperature, $\Delta E_{\text{int}} = \boxed{0 \text{ J}}$

- (c) The heat flow can be found from the first law of thermodynamics.

$$\Delta E_{\text{int}} = Q - W \rightarrow Q = \Delta E_{\text{int}} + W = 0 + 480 \text{ J} = \boxed{480 \text{ J (into the gas)}}$$

- (b) [12 marks]. A 3.65-mol sample of an ideal diatomic gas expands adiabatically from a volume of 0.1210 m^3 to 0.750 m^3 . Initially the pressure was 1.00 atm. Determine: (i) the initial and final temperatures; (ii) the change in internal energy; (iii) the heat lost by the gas; (iv) the work done on the gas.

Answer:

- (a) We first find the final pressure from the adiabatic relationship, and then use the ideal gas law to find the temperatures. For a diatomic gas, $\gamma = 1.4$.

$$P_1 V_1^\gamma = P_2 V_2^\gamma \rightarrow P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = (1.00 \text{ atm}) \left(\frac{0.1210 \text{ m}^3}{0.750 \text{ m}^3} \right)^{1.4} = 7.772 \times 10^{-2} \text{ atm}$$

$$PV = nRT \rightarrow T = \frac{PV}{nR} \quad T_1 = \frac{P_1 V_1}{nR} = \frac{(1.013 \times 10^5 \text{ Pa})(0.1210 \text{ m}^3)}{(3.65 \text{ mol}) \left(8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \right)} = 403.9 \text{ K} \approx \boxed{404 \text{ K}}$$

$$T_2 = \frac{P_2 V_2}{nR} = \frac{(7.772 \times 10^{-2}) (1.013 \times 10^5 \text{ Pa})(0.750 \text{ m}^3)}{(3.65 \text{ mol}) \left(8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \right)} = 194.6 \text{ K} \approx \boxed{195 \text{ K}}$$

$$(b) \Delta E_{\text{int}} = nC_c\Delta T = n\left(\frac{5}{2}R\right)\Delta T = (3.65 \text{ mol})\frac{5}{2}\left(8.314 \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(194.6 \text{ K} - 403.9 \text{ K}) = \boxed{-1.59 \times 10^4 \text{ J}}$$

(c) Since the process is adiabatic, no heat is transferred. $\boxed{Q = 0}$

(d) Use the first law of thermodynamics to find the work done by the gas. The work done ON the gas is the opposite of the work done BY the gas.

$$\Delta E_{\text{int}} = Q - W \rightarrow W = Q - \Delta E_{\text{int}} = 0 - (-1.59 \times 10^4 \text{ J}) = 1.59 \times 10^4 \text{ J}$$

$$W_{\text{on gas}} = \boxed{-1.59 \times 10^4 \text{ J}}$$

Question ④

《18 marks》

(a) [6 marks] A 2.00 mol monatomic gas initially at 300 K undergoes this cycle: It is (1) heated at constant volume to 800 K, (2) then allowed to expand isothermally to its initial pressure, (3) then compressed at constant pressure to its initial state. Find the net work done by the gas.

Answer:

$$a) W = nRT \ln\left(\frac{V_3}{V_2}\right) + nR(T_3 - T_1)$$

$$\text{For isothermal process } 2 \rightarrow 3: \left(\frac{V_3}{V_2}\right) = \left(\frac{P_2}{P_3}\right) = \left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)$$

$$W = 2 * 8.314 * 800 \ln\left(\frac{800}{300}\right) + 2 * 8.314(300 - 800) = 4730 \text{ J}$$

(b) [6 marks] A Carnot engine has a power of 500 W. It operates between heat reservoirs at 100 °C and 60.0 °C. Calculate (i) the rate of heat input and (ii) the rate of exhaust heat output.

Answer:

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{333}{373} = 0.107$$

$$\frac{Q_H}{\Delta t} = \frac{1 \text{ W}}{\varepsilon \Delta t} = \frac{1}{0.107} * 500 = 4672.9 \text{ W}$$

$$\frac{Q_C}{\Delta t} = \frac{Q_H}{\Delta t} - \frac{W}{\Delta t} = 4172.9 \text{ W}$$

[6 marks] The tungsten filament of a light bulb has an operating temperature of about 2100 K. If the emitting area of the filament is 1.0 cm², and its emissivity is 0.68, what is the power output of the light bulb?

Answer:

$$P = A\varepsilon\sigma T^4 = 1 * 10^{-4} * 0.68 * 5.67 * 10^{-8} * 2100^4 = 75 \text{ W}$$