Benha University | **Department:** Electrical Engineering
Benha Faculty of Engineering | **Time:** 3 Hours.
2nd **Term,** 2020/2021 | **BSc. 4th Year**
Subject: Digital Image Processing and Pattern Recognition [E1528]
Examiner: *Dr. Ayman Soliman*

No. of pages: 4 | **No. of questions: 5** | **Total marks: 90**

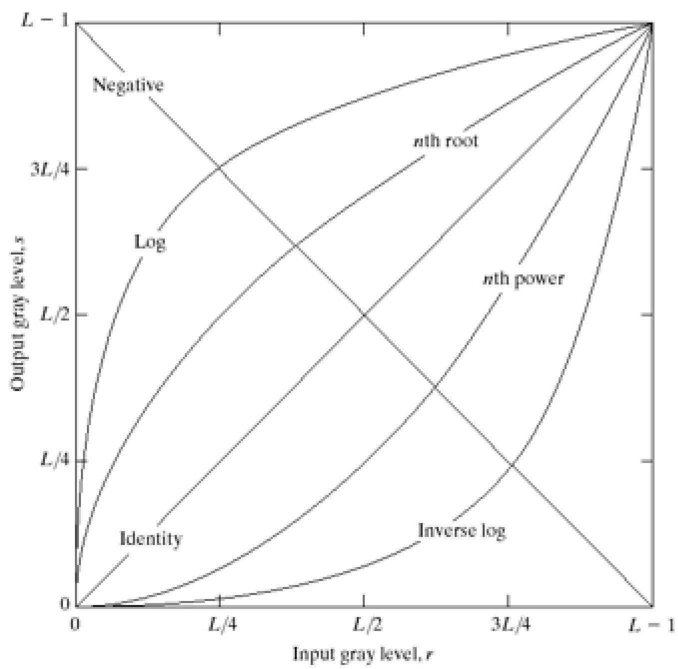
Question ❶ (30 marks) (1 mark for each point)

Choose the correct answer:

1	2	3	4	5	6	7	8	9	10
c	B	d	c	b	b	a	a	c	b
11	12	13	14	15	16	17	18	19	20
c	b	a	a	a	d	c	a	b	c
21	22	23	24	25	26	27	28	29	30
b	d	c	c	b	a	b	d	b	c

Question ❷ (20 marks) ((1) 5 marks – (2) 5 marks – (3) 10 marks)

1) Explain with drawing the intensity transformation functions? What is the problem arising from using full scale contrast stretching?



Negative:
$s = L - 1 - r$

Log:
$s = c\log(1 + r)$

Inverse Log:
$s = e^{cr} - 1$

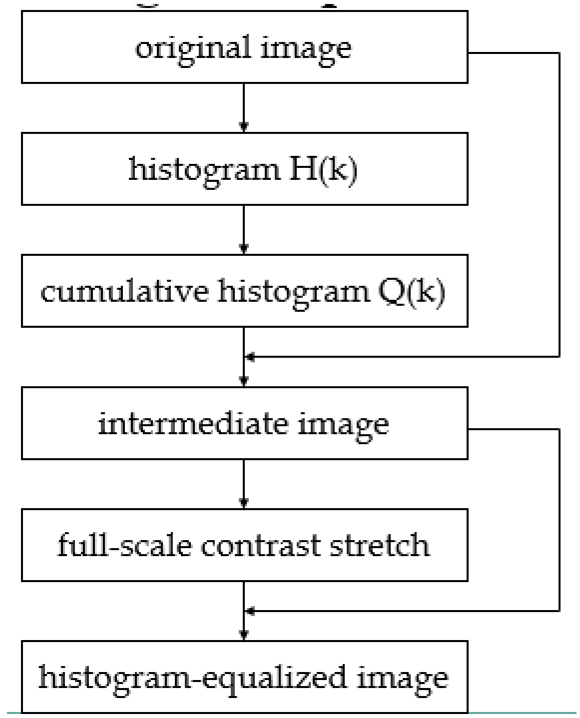
Power-law:
$s = cr^{\gamma}$

……

Then explain intensity transformation functions

The major problem arising from using full scale contrast stretching when using an image with big gap which will increase using this technique.

2) Draw and explain the block diagram of the histogram equalization algorithm.



Then explain the sequence of block diagram

3) For a 4x4, 4bits/pixel image shown.

- Find the full-scale contrast stretched image.
- Find the histogram equalized image.
- Draw the histogram of the three images and compare between them.

2	8	9	9
2	3	10	9
8	3	3	11
8	3	10	11

(Hint: the equation of contrast stretch is $s = round\left((2^B - 1) \cdot \frac{r - r_{\min}}{r_{\max} - r_{\min}} \right)$)

- First try: full-scale contrast stretch $r_{\min} = 2$ $r_{\max} = 11$

$$s = round\left((2^B - 1) \cdot \frac{r - r_{\min}}{r_{\max} - r_{\min}} \right) = round\left(15 \cdot \frac{r-2}{11-2} \right) = round\left(\frac{5}{3}(r-2) \right)$$

2 → round(0) = 0;
3 → round(1.67) = 2;
8 → round(10.00) = 10;
9 → round(11.67) = 12;
10 → round(13.33) = 13;
11 → round(15) = 15;

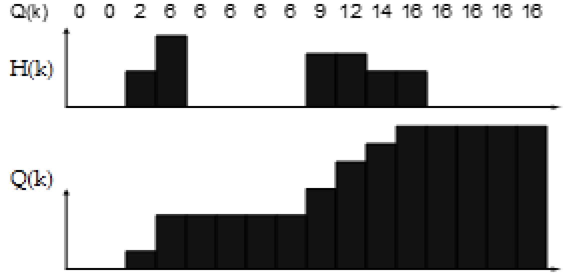
The resulting image is:

0	10	12	12
0	2	13	12
10	2	2	15
10	2	13	15

Cumulative Histogram

2	8	9	9
2	3	10	9
8	3	3	11
8	3	10	11

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	0	0	2	4	0	0	0	0	3	3	2	2	0	0	0	0
Q(k)	0	0	2	6	6	6	6	6	9	12	14	16	16	16	16	16



Intermediate Image

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	0	0	2	4	0	0	0	0	3	3	2	2	0	0	0	0
Q(k)	0	0	2	6	6	6	6	6	9	12	14	16	16	16	16	16

original

2	8	9	9
2	3	10	9
8	3	3	11
8	3	10	11

→

2	9	12	12
2	6	14	12
9	6	6	16
9	6	14	16

intermediate image

Full-Scale Contrast Stretch of Intermediate Image

intermediate image

2	9	12	12
2	6	14	12
9	6	6	16
9	6	14	16

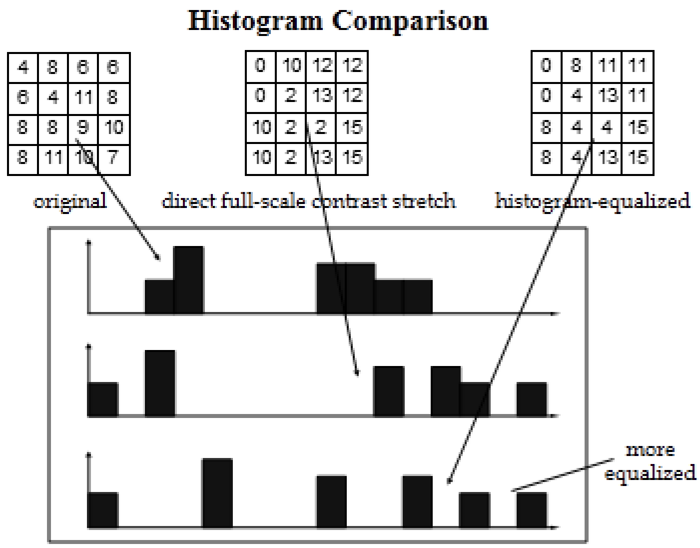
$r_{min} = 2 \qquad r_{max} = 16$

$$s = round\left((2^B - 1)\cdot\frac{r - r_{min}}{r_{max} - r_{min}}\right) = round\left(15\cdot\frac{r-2}{16-2}\right) = round\left(\frac{15}{14}(r-2)\right)$$

2 → round(0) = 0;
6 → round(4.29) = 4;
9 → round(7.50) = 8;
12 → round(10.71) = 11;
14 → round(12.86) = 13;
16 → round(15) = 15;

final result: histogram equalized image

0	8	11	11
0	4	13	11
8	4	4	15
8	4	13	15



Question ❸ (10 marks) (2.5 marks for each point)

1) What is Padding? Briefly explain the **three** types of padding?

   Padding extends the boundaries of an image to avoid undefined operations when parts of a kernel (mask) lie outside the border of the image during filtering. In general, if the kernel is of size m*n, we need (m-1)/2 rows at the top and bottom and (n-1)/2 columns at the right and left and the values these new cells will take depends on the padding technique.

Types

- Zero padding
- Mirror (or symmetric) padding
- Replicate padding

Zero padding put zeros around all border image

0	0	0	0	0	0
0	1	8	6	6	0
0	6	3	11	8	0
0	8	8	9	10	0
0	9	10	10	7	0
0	0	0	0	0	0

Mirror padding

Values outside the boundary of the image are obtained by mirror-reflecting the image across its border.

13	12	11	12	13	14	15	14	13
8	7	6	7	8	9	10	9	8
3	2	1	2	3	4	5	4	3
8	7	6	7	8	9	10	9	8
13	12	11	12	13	14	15	14	13
18	17	16	17	18	19	20	19	18
13	12	11	12	13	14	15	14	13
8	7	6	7	8	9	10	9	8

Replicate padding

Values outside the boundary are set equal to the nearest image border value. It is useful when the areas near the border of the image are constant.

1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
6	6	6	7	8	9	10	10	10
11	11	11	12	13	14	15	15	15
16	16	16	17	18	19	20	20	20
16	16	16	17	18	19	20	20	20
16	16	16	17	18	19	20	20	20

Question ❹ (20 marks) (2.5 marks for each point)

For a 4x4, 4bits/pixel image shown.

1) Compute its 2D-DFT
2) lowest frequency component
3) highest frequency component
4) Real part
5) Imaginary part
6) Magnitude
7) Phase
8) Compute the inverse 2D-DFT

$$X = \begin{bmatrix} 1 & 3 & 6 & 8 \\ 9 & 8 & 8 & 2 \\ 5 & 4 & 2 & 3 \\ 6 & 6 & 3 & 3 \end{bmatrix}$$

hint: $\tilde{X} = F_4 X F_4$, $\quad X = \frac{1}{N^2} F_N^* \tilde{X} F_N^*$

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \qquad F_4^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$\mathrm{X} = \begin{bmatrix} 1 & 3 & 6 & 8 \\ 9 & 8 & 8 & 2 \\ 5 & 4 & 2 & 3 \\ 6 & 6 & 3 & 3 \end{bmatrix} \qquad \tilde{\mathrm{X}} = \mathrm{F_4 X F_4} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 & 3 & 6 & 8 \\ 9 & 8 & 8 & 2 \\ 5 & 4 & 2 & 3 \\ 6 & 6 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

MATLAB function: *fft2*

$$= \begin{bmatrix} 21 & 21 & 19 & 16 \\ -4-3j & -1-2j & 4-5j & 5+j \\ -9 & -7 & -3 & 6 \\ -4+3j & -1+2j & 4+5j & 5-j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

lowest frequency component

$$= \begin{bmatrix} 77 & 2-5j & 3 & 2+5j \\ 4-9j & -11+8j & -4-7j & -5-4j \\ -13 & -6+13j & -11 & -6-13j \\ 4+9j & -5+4j & -4+7j & -11-8j \end{bmatrix}$$

highest frequency component

Real part:

$$\tilde{\mathrm{X}}_{real} = \begin{bmatrix} 77 & 2 & 3 & 2 \\ 4 & -11 & -4 & -5 \\ -13 & -6 & -11 & -6 \\ 4 & -5 & -4 & -11 \end{bmatrix}$$

Imaginary part:

$$\tilde{\mathrm{X}}_{imag} = \begin{bmatrix} 0 & -5 & 0 & 5 \\ -9 & 8 & -7 & -4 \\ 0 & 13 & 0 & -13 \\ 9 & 4 & 7 & -8 \end{bmatrix}$$

Magnitude:

$$\tilde{\mathrm{X}}_{magnitude} = \begin{bmatrix} 77 & 5.39 & 3 & 5.39 \\ 9.85 & 13.60 & 8.06 & 6.4 \\ 13 & 14.32 & 11 & 14.32 \\ 9.85 & 6.40 & 8.06 & 13.60 \end{bmatrix}$$

Phase:

$$\tilde{\mathrm{X}}_{phase} = \begin{bmatrix} 0 & -1.19 & 0 & 1.19 \\ -1.15 & 2.51 & -2.09 & -2.47 \\ 3.14 & 2.00 & 3.14 & -2.00 \\ 1.15 & 2.47 & 2.09 & -2.51 \end{bmatrix}$$

- Compute the inverse 2D-DFT:

$$F_4^* \tilde{X} F_4^* = \frac{1}{4^2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & -j & -1 & j\end{bmatrix}\begin{bmatrix}77 & 2-5j & 3 & 2+5j\\ 4-9j & -11+8j & -4-7j & -5-4j\\ -13 & -6+13j & -11 & -6-13j\\ 4+9j & -5+4j & -4+7j & -11-8j\end{bmatrix}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & -j & -1 & j\end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & -j & -1 & j\end{bmatrix}\begin{bmatrix}21 & 21 & 19 & 16\\ -4-3j & -1-2j & 4-5j & 5+j\\ -9 & -7 & -3 & 6\\ -4+3j & -1+2j & 4+5j & 5-j\end{bmatrix}$$

$$= \begin{bmatrix}1 & 3 & 6 & 8\\ 9 & 8 & 8 & 2\\ 5 & 4 & 2 & 3\\ 6 & 6 & 3 & 3\end{bmatrix} = X$$

<u>Question ❺ (10 marks)</u>

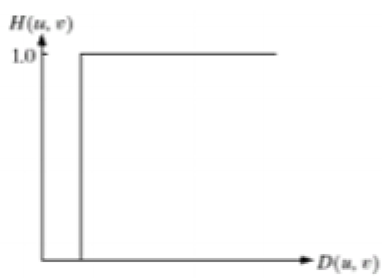
Compare between

- Ideal high pass filter
- Butterworth high pass filter
- Gaussian high pass filter

Ideal high pass filter

The ideal high pass filter is given as:

$$H(u,v) = \begin{cases} 0 \text{ if } D(u,v) \le D_0 \\ 1 \text{ if } D(u,v) > D_0 \end{cases}$$



where D_0 is the cut off. means that any value smaller than the D_0 will be ignored and any value bigger than D_0 will be counted

Butterworth high pass filter

This Butterworth highpass filter is the reverse operation of the Butterworth lowpass filter. It can be determined using the relation- $H_{HP}(u,v) = 1 - H_{LP}(u,v)$ where, $H_{HP}(u,v)$ is the transfer function of the highpass filter and $H_{LP}(u,v)$ is the transfer function of the corresponding lowpass filter.

The transfer function of BHPF of order n is defined as-

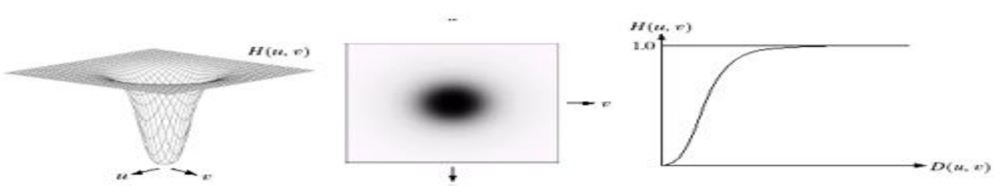
$$H(u,v) = \frac{1}{1+[D_0/D(u,v)]^{2n}}$$

- D_0 is a positive constant. BHPF passes all the frequencies greater than D_0 value without attenuation and cuts off all the frequencies less than it.

- D_0 This is the transition point between H(u, v) = 1 and H(u, v) = 0, so this is termed as *cutoff frequency*. But instead of making a sharp cut-off (like, Ideal Highpass Filter (IHPF)), it introduces a smooth transition from 0 to 1 to reduce ringing artifacts. • $D(u,v)$ is the Euclidean Distance from any point (u, v) to the origin of the frequency plane, i.e. $D(u,v) = \sqrt{(u^2 + v^2)}$,

The Butterworth high pass filter is given as:

$$H(u,v) = \frac{1}{1+[D_o / D(u,v)]^{2n}}$$



Gaussian High Pass Filters

The Gaussian high pass filter is given as:

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

where D_0 is the cut off distance as before